

# **STUDENT SUPPORT MATERIAL**

## **Class X**

### **Mathematics**



**Session 2016-17**

**KENDRIYA VIDYALAYA SANGATHAN**  
**NEW DELHI**

# **STUDENT SUPPORT MATERIAL**

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## INDEX

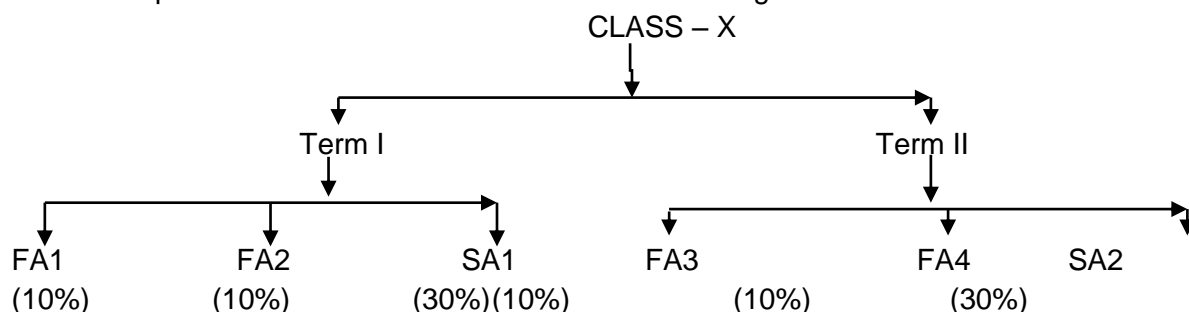
SL.NO	TOPIC
	<b>SA-1 PART -1</b>
1	Real Numbers
2	Polynomials
3	A pair of linear equations in two variables
4	Triangles
5	Introduction to Trigonometry
6	Statistics
7	Model Question Paper SA-1
	<b>PART – 2</b>
8	Activities (Term I)

SLNO	TOPIC
	<b>SA- 2 PART - 1</b>
1	Quadratic Equation
2	Arithmetic Progression
3	Coordinate Geometry
4	Some Applications of Trigonometry
5	Circle
6	Construction
7	Area Related to Circle
8	Surface Area and Volume
9	Probability
10	Model Question paper SA-2
	<b>PART – 2</b>
11	Activities (Term II)

## COURSE STRUCTURE

### CLASS –X

As per CCE guidelines, the syllabus of Mathematics for class X has been divided term-wise. The units specified for each term shall be assessed through both formative and summative assessment.



Suggested activities and projects will necessarily be assessed through formative assessment.

### SUMMATIVE ASSESSMENT -I

S.NO	TOPIC	MARKS: 90
<b>SA-I</b>		
<b>1</b>	<b>NUMBER SYSTEM</b> Real Numbers	<b>11</b>
<b>2</b>	<b>ALGEBRA</b> Polynomials, pair of linear equations in two variables.	<b>23</b>
<b>3</b>	<b>GEOMETRY</b> Triangles	<b>17</b>
<b>4</b>	<b>TRIGONOMETRY</b> Introduction to trigonometry, trigonometric identity.	<b>22</b>
<b>5</b>	<b>STATISTICS</b>	<b>17</b>
	TOTAL	90

## **TOPIC WISE ANALYSIS OF EXAMPLES AND QUESTIONS**

### **NCERT TEXT BOOK**

Chapters	Topics	Number of Questions for revision		Total
		Questions from solved examples	Questions from exercise	
1	Real Number	11	18	29
2	Polynomials	09	08	17
3	Pair of linear equations in two variables	19	21	40
4	Triangles	14	55	69
5	Introduction to trigonometry	15	27	42
6	Statistics	09	25	34
Total		77	144	231

**DETAILS OF THE CONCEPTS TO BE MASTERED BY EVERY CHILD OF CLASS X WITH  
EXERCISE AND EXAMPLES OF NCERT TEXT BOOKS.**

**SA - I**

**SYMBOLS USED**

TG/LG is idea identified by termwise error analysis of answers of Q.P. of SA of last three year.

\* - Important Question

a - Low

T.G-Teaching Gap

\*\* -Very Important Question

b - Average

L.G-Learning Gap

\*\*\* -Very Very Important Question

c - Higher

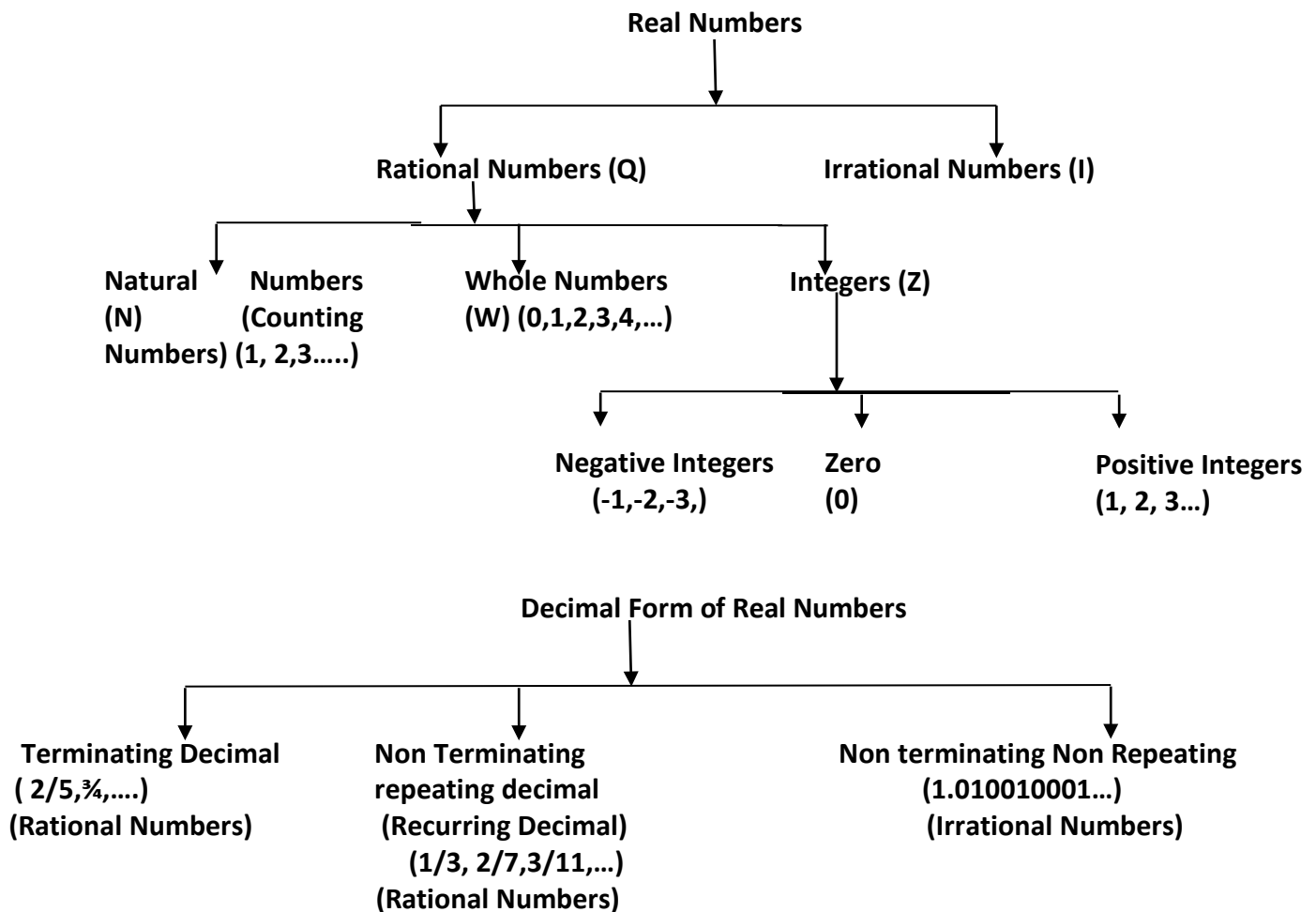
S.No	TOPIC	CONCEPTS	DEGREE OF IMPORTANCE	DIFFICULTY LEVEL		REFERENCES(NCER T BOOK)
				TG/LG	DEGREE	
01	Real Number	Euclid's division Lemma & Algorithm	***	L.G	a	Example -1,2,3,4 Ex:1.1 Q:1,2,4
		Fundamental Theorem of Arithmetic	***	L.G	a	Example -5,7,8 Ex:1.2 Q:4,5
		Revisiting Irrational Numbers	***	L.G	b	Example -9,10,11 Ex: 1.3 Q:1.2 Th:1.4
		Revisiting Rational Number and their decimal Expansion	**	L.G	a	Ex -1.4 Q:1
02	Polynomials	Meaning of the zero of Polynomial	*	L.G	a	Ex -2.1 Q:1
		Relationship between zeroes and coefficients of a polynomial	**	L.G	a	Example -2,3 Ex-2.2 Q:1
		Forming a quadratic polynomial	**	L.G	b	Ex -2.2 Q:2
		Division algorithm for a polynomial	*	L.G	b	Ex -2.3 Q:1,2
		Finding the zeroes of a polynomial	***	L.G	a	Example: 9 Ex -2.3 Q:1,2,3,4,5 Ex-2.4,3,4,5
03	Pair of Linear Equations in two variables	Graphical algebraic representation	*	L.G	b	Example:2,3 Ex -3.4 Q:1,3
		Consistency of pair of liner equations	**	L.G	a	Ex -3.2 Q:2,4
		Graphical method of solution	***	L.G	b	Example: 4,5 Ex -3.2 Q:7
		Algebraic methods of solution a. Substitution method b. Elimination method c. Cross multiplication method d. Equation L.G reducible to pair of liner equation in two variables	**	L.G	b	Ex -3.3 Q:1,3  Example-13 Ex:3.4 Q:1,2  Example-15,16 Ex:3.5 Q:1,2,4  Example-19 Ex-3.6

						Q :1(ii),(viii),2 (ii),(iii)
04	Triangles	1) Similarity of Triangles	***	T.G	C	Theo:6.1 Example:1,2,3 Ex:6.2 Q:2,4,6,9,10
		2) Criteria for Similarity of Triangles	**	T.G	C	Example:6,7 Ex:6.3 Q:4,5,6,10,13,16
		3) Area of Similar Triangles	***	L.G	B	Example:9 The:6.6 Ex:6.4 Q:3,5,6,7
		4) Pythagoras Theorem	***	L.G	b	Theo:6.8 & 6.9 Example:10,12,14, Ex:6.5 Q:4,5,6,7,13,14,15,16
05	Introduction to Trigonometry	1) Trigonometric Ratios	*	L.G	a	Ex:8.1 Q:1,2,3,6,8,10
		2) Trigonometric ratios of some specific angles	**	L.G	b	Example:10,11 Ex:8.2 Q:1,3
		3) Trigonometric ratios of complementary angles	**	L.G	a	Example:14,15 Ex:8.3 Q:2,3,4,6
		4) Trigonometric Identities	***	L.G	b	Ex:8.4 Q:5 (iii,v,viii)

06	Statistics	CONCEPT 1 Mean of grouped data	***	L.G	a	
		1. Direct Method	*	L.G	b	Example:2 Ex:14.1 Q:1&3
		2. Assumed Mean Method	*	L.G	b	Ex:14.1 Q:6
		3. Step Deviation Method		L.G	b	Ex:14.1 Q:9
		CONCEPT 2	***	L.G		
		Mode of grouped data		L.G	a	Example:5 Ex:14.2 Q:1,5
		CONCEPT 3	***	L.G		
		Median of grouped data		L.G	a	Example:7,8 Ex:14.3 Q1,3,5
		CONCEPT 4	**	L.G		
		Graphical representation of c.f.(give)	***	L.G	b	Example:9 Ex:14.4 Q:1,2,3

# Real Numbers

(Key Points)



- Euclid's Division lemma:-** Given Positive integers  $a$  and  $b$  there exist unique integers  $q$  and  $r$  satisfying  $a = bq + r$ , where  $0 \leq r < b$ , where  $a$ ,  $b$ ,  $q$  and  $r$  are respectively called as dividend, divisor, quotient and remainder.
- Euclid's division Algorithm:-** To obtain the HCF of two positive integers say  $c$  and  $d$ , with  $c > d$ , follow the steps below:
  - Step I:** Apply Euclid's division lemma, to  $c$  and  $d$ , so we find whole numbers,  $q$  and  $r$  such that  $c = dq + r$ ,  $0 \leq r < d$ .
  - Step II:** If  $r = 0$ ,  $d$  is the HCF of  $c$  and  $d$ . If  $r \neq 0$ , apply the division lemma to  $d$  and  $r$ .
  - Step III:** Continue the process till the remainder is zero. The divisor at this stage will be the required HCF

Note:- Let  $a$  and  $b$  be positive integers. If  $a = bq + r$ ,  $0 \leq r < b$ , then  $\text{HCF}(a, b) = \text{HCF}(b, r)$

### 3. The Fundamental theorem of Arithmetic:-

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex.:  $24 = 2 \times 2 \times 2 \times 3 = 3 \times 2 \times 2 \times 2$



**Theorem:** Let  $x$  be a rational number whose decimal expansion terminates. Then  $x$  can be expressed in the form

Of  $p/q$  where  $p$  and  $q$  are co-prime and the prime factorization of  $q$  is of the form of  $2^n \cdot 5^m$ , where  $n, m$  are non-negative integers.

Ex.  $\frac{7}{10} = \frac{7}{2 \times 5} = 0.7$

**Theorem:** Let  $x = \frac{p}{q}$  be a rational number such that the prime factorization of  $q$  is not of the form of  $2^n \cdot 5^m$ , where  $n, m$  are non-negative integers. Then  $x$  has a decimal expansion which is non-terminating repeating (recurring).

Ex.  $\frac{7}{6} = \frac{7}{2 \times 3} = 1.1666 \dots$

**Theorem:** For any two positive integers  $a$  and  $b$ , HCF

$(a, b) \times \text{LCM}(a, b) = a \times b$

Ex.:  $4 \times 6$ ; HCF  $(4, 6) = 2$ , LCM  $(4, 6) = 12$ ; HCF  $\times$  LCM  $= 2 \times 12 = 24$

Ans.:  $a \times b = 24$

## LEVEL-I

1. If  $\frac{p}{q}$  is a rational number ( $q \neq 0$ ). What is the condition on  $q$  so that the decimal representation of  $\frac{p}{q}$  is terminating?
2. Write a rational number between  $\sqrt{2}$  and  $\sqrt{3}$ .
3. The decimal expansion of the rational number  $\frac{43}{2^4 5^3}$  will terminate after how many places of decimal?
4. Find the  $(\text{HCF} \times \text{LCM})$  for the numbers 100 and 190.
5. State whether the number  $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$  is rational or irrational justify.
6. Write one rational and one irrational number lying between 0.25 and 0.32.
7. Express 107 in the form of  $4q+3$  for some positive integer  $q$ .
8. Write whether the rational number  $\frac{51}{1500}$  will have a terminating decimal expansion or a non-terminating repeating decimal expansion.
9. Show that any positive odd integer is of the form  $6q+1$  or  $6q+3$  or  $6q+5$ , where  $q$  is some integer.
10. Express 0.2545454..... As a fraction in simplest form.

## LEVEL-II

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.
2. Check whether  $5 \times 3 \times 11 + 11$  and  $5 \times 7 + 7 \times 3$  are composite numbers and justify.
3. Check whether  $6^n$  can end with the digit 0, where  $n$  is any natural number.

4. Given that  $\text{LCM}(26, 169) = 338$ , write  $\text{HCF}(26, 169)$ .]
5. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.
6. Use Euclid's division lemma to show that the square of any positive integer is either of the form  $3m$  or  $3m+1$  for some integer  $m$ .
7. Use Euclid's division lemma to show that the cube of any positive integer is of the form  $9m$ ,  $9m+1$  or  $9m+8$  for some integer  $m$ .

### **LEVEL-III**

1. Show that  $\sqrt{3}$  is an irrational number.
2. Show that  $5 + 3\sqrt{2}$  is an irrational number.
3. Show that square of an odd positive integer is of the form  $8m+1$ , for some integer  $m$ .
4. Find the LCM & HCF of 26 and 91 and verify that
5. Prove that  $\sqrt[3]{7}$  is irrational.
6. Show that one and only one out of  $n$ ,  $n+2$ ,  $n+4$  is divisible by 3, where  $n$  is any positive integer.
7. Find the HCF of 65 & 117 and express it in the form of  $65m + 117n$ .

### **(PROBLEMS FOR SELF EVALUATION/HOTS)**

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Find the LCM and HCF of 17, 23 and 29.
4. Prove that  $\sqrt{2}$  is not a rational number.
5. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.
6. Show that there is no positive integer  $n$  for which  $\sqrt{n-1} + \sqrt{n+1}$  is rational.
7. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that  
HCF  $\times$  LCM  $\neq$  product of three numbers.
8. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. How many stacks will be there?

## Value Based Questions

Q.1 A person wanted to distribute 96 apples and 112 oranges among poor children in an orphanage. He packed all the fruits in boxes in such a way that each box contains fruits of the same variety, and also every box contains an equal number of fruits.

(i) Find the maximum number of boxes in which all the fruits can be packed.

(ii) Which concept have you used to find it?

(iii) Which values of this person have been reflected in above situation?

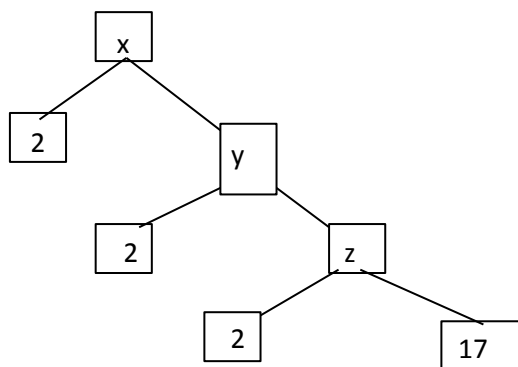
Q.2 A teacher draws the factor tree given in figure and ask the students to find the value of  $x$  without finding the value of  $y$  and  $z$ .

Shaurya gives the answer  $x=136$

a) Is his answer correct?

b) Give reason for your answer.

c) Which value is depicted in this?



## **Answer**

### **Level-I**

1.  $q$  is of the form  $2^n .5^m$ , where  $m$  and  $n$  are non-negative integers.
2. 1.5
3. After 4 places of decimal.
4. 19000
5. Rational number
6. One rational number= $26/100$ , one irrational no.= $0.27010010001\dots\dots$
7.  $4 \times 26+3$
8. Terminating
10.  $14/55$

### **Level-II**

- 1.23
2. Composite number
3. No,  $6^n$  cannot end with the digit 0.
- 4.13
5.  $HCF=6$  ,  $LCM = 360$

### **Level-III**

4.  $LCM= 182$  , $HCF = 13$
7.  $m = 2$  and  $n = -1$ .

### **Problems for self-evaluation**

1. See textbook.
2.  $2658 = 2 \times 3 \times 443$
3.  $HCF = 1$  ,  $LCM = 11339$
5. 13
8. Total no. of stacks = 14

### **Value based Questions**

1. (i)No. of boxes = 16  
(ii)Number System & HCF  
(iii)The person is kind hearted and of helping attitude.
2. (a) Yes, his answer is correct.  
(b)  $Z = 2 \times 17 = 34$ ,  $Y = 2 \times 34 = 68$ ,  $X = 2 \times 68 = 136$   
(c) Knowledge of prime factorization.

# Polynomial

## Polynomial

An expression of the form  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$  where  $a_n \neq 0$  is called a polynomial in one variable  $x$  of degree  $n$ , where;  $a_0, a_1, a_2, \dots, a_n$  are constants and they are called the coefficients of  $x_0, x, x^2, \dots, x^n$ . Each power of  $x$  is a non-negative integer.

Eg:  $-2x^2 - 5x + 1$  is a polynomial of degree 2

Note:  $\sqrt{x} + 3$  is not a polynomial

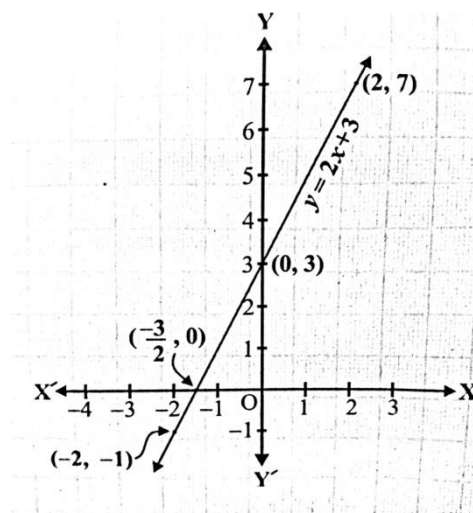
- A polynomial  $p(x) = ax + b$  of degree 1 is called a linear polynomial Eg:  $5x - 3, 2x$  etc
- A polynomial  $p(x) = ax^2 + bx + c$  of degree 2 is called a quadratic polynomial Eg:  $2x^2 + x - 1$
- A polynomial  $p(x) = ax^3 + bx^2 + cx + d$  of degree 3 is called a cubic polynomial.  
Eg:  $\sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1$  etc

Zeros of a polynomial: A real number  $k$  is called a zero of polynomial  $p(x)$  if  $p(k)=0$ . If the graph of  $y= p(x)$  intersects the  $X$ -axis at  $n$  times, the number of zeroes of  $y= p(x)$  is  $n$ .

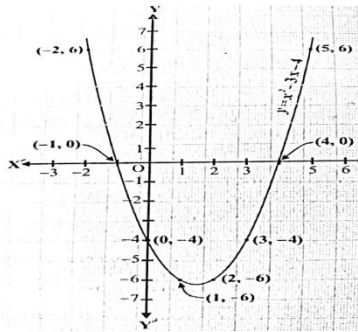
- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

### Graphs of different types of polynomials:

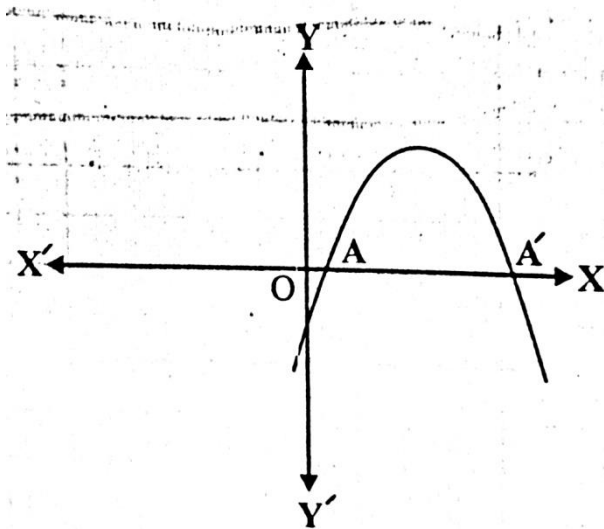
- Linear polynomial:- The graph of a linear polynomial  $ax+b$  is a straight line, intersecting  $X$ - axis at one point



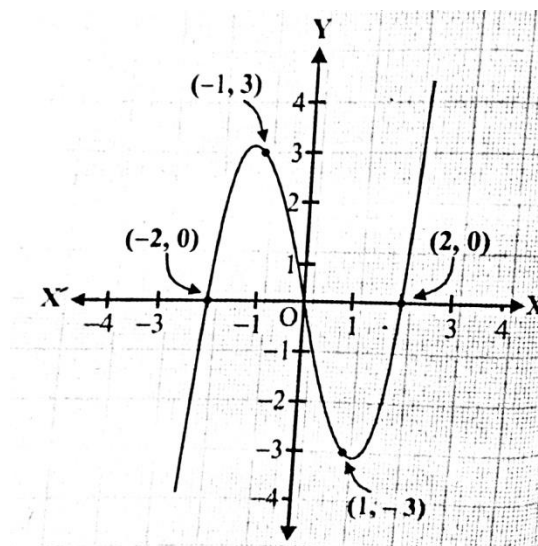
- Quadratic polynomial:-  
(i) Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola open upwards like U, if  $a > 0$  & intersects  $x$ -axis at maximum two distinct points.



(ii) Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola open downwards like  $\cap$  if  $a < 0$  & intersects x-axis at maximum two distinct points



- Cubic polynomial and its graph:- in general a polynomial  $p(x)$  of degree  $n$  crosses the x-axis at most  $n$  points.



For a quadratic polynomial:- If  $\alpha, \beta$  are zeroes of  $p(x) = ax^2 + bx + c$  then,

1. Sum of zeroes  $= \alpha + \beta = -\frac{b}{a} = \frac{\text{-coefficients of } x}{\text{coefficient of } x^2}$
2. Product of zeroes  $= \alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

- A quadratic polynomial whose zeroes are  $\alpha$  and  $\beta$ , is given by:

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

- If  $\alpha, \beta$  and  $\gamma$  are zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  then:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

- If  $\alpha, \beta, \gamma$  are zeroes of a cubic polynomial  $p(x)$ ,

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

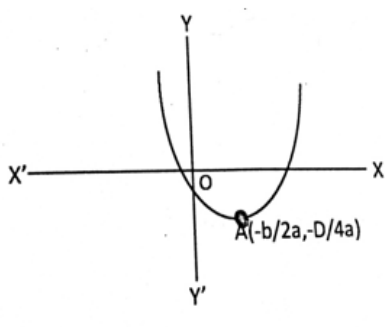
**Division algorithm for polynomials:** If  $p(x)$  and  $g(x)$  are any two polynomials with  $g(x) \neq 0$ , then we have polynomials  $q(x)$  and  $r(x)$  such that

$$P(x) = g(x) \times q(x) + r(x), \quad \text{where } r(x) = 0 \text{ or degree of } r(x) < \text{degree of } g(x).$$

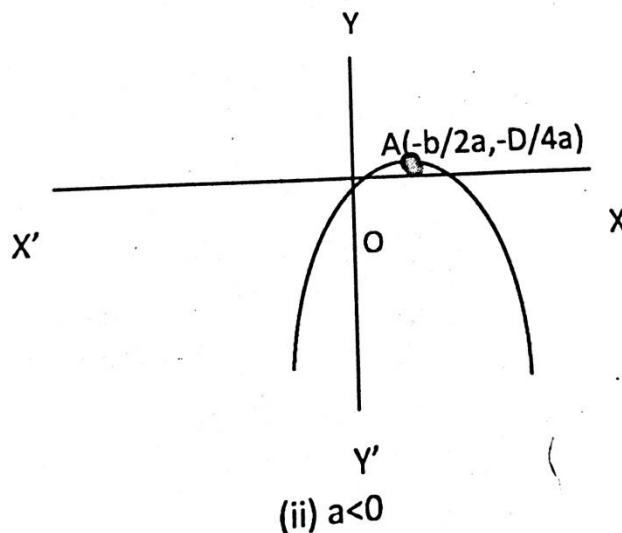
**Nature of graph of polynomial  $P(x) = ax^2 + bx + c$ :-**

Case-1 When polynomial  $ax^2 + bx + c$  is factorable in two distinct linear factors.

In this case, curve cuts X-axis at two distinct points. The co-ordinate of the vertex of parabola are  $(-b/2a, -D/4a)$  where  $D = b^2 - 4ac$ . The x co-ordinates of these points are the two zeroes of the polynomial.



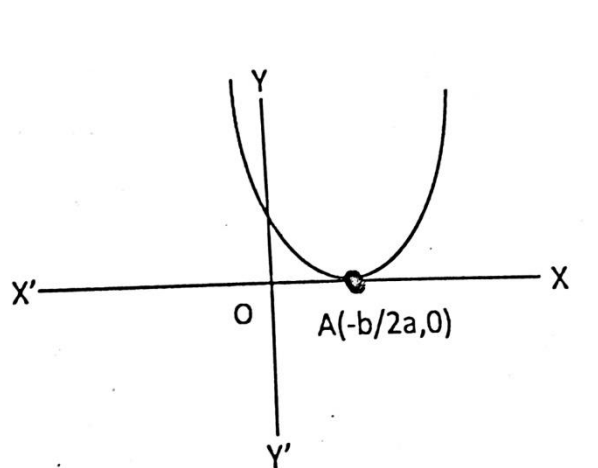
(i)  $a > 0$



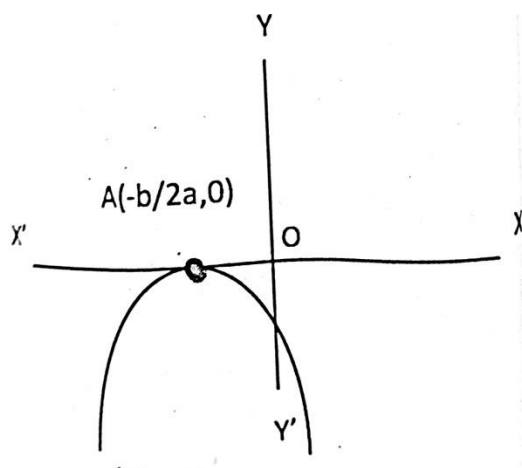
(ii)  $a < 0$

Case 2:- When Polynomial  $ax^2 + bx + c$  is factorisable into two equal factors.

In this case, curve touches X-axis at the point  $(-b/2a, 0)$ . The x- Co-ordinates of the point gives two equal zeroes of the polynomial.

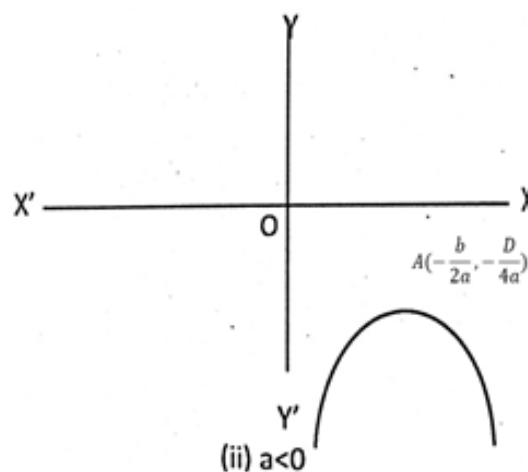
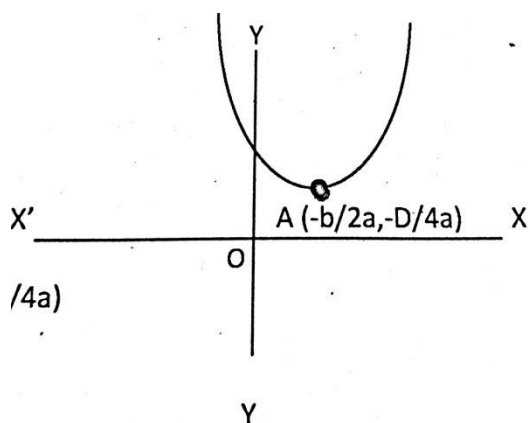


(i)  $a > 0$



(ii)  $a < 0$

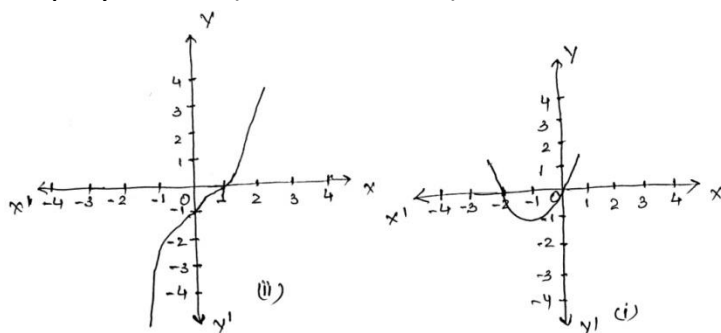
Case- 3 When Polynomial  $ax^2 + bx + c$  is not factorizable. In this case, the curve doesn't cut or touches X-axis



(ii)  $a < 0$

## Level – I

- Find the value of zeroes of the polynomials  $p(x)$  as shown in the graph and hence find the polynomial. (CBSE 2014-15).



- Let  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial  $2x^2 - 5x - 6$  then form a quadratic polynomial whose zeroes are  $\alpha + \beta$  and  $\alpha\beta$ . (CBSE 2011)



3. Check whether  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ ?  
(CBSE 2010)
4. Can  $(x-7)$  be the remainder on division of a polynomial  $p(x)$  by  $(7x + 2)$ ? Justify your answer(CBSE 2010)
5. What must be subtracted from the polynomial  $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ , so that the resulting polynomial is exactly divisible by  $x^2 - 4x + 3$ ? (CBSE 2013)
6. Write the degree of zero polynomial?
7. Find the zeroes of a quadratic polynomial  $6x^2 - 7x - 3$  and verify the relationship between the zeroes and the coefficients? (CBSE 2014-15)
8. Find the quadratic polynomial sum of whose zeroes is  $2\sqrt{3}$  and their product is 2?(CBSE 2008)

## Level II

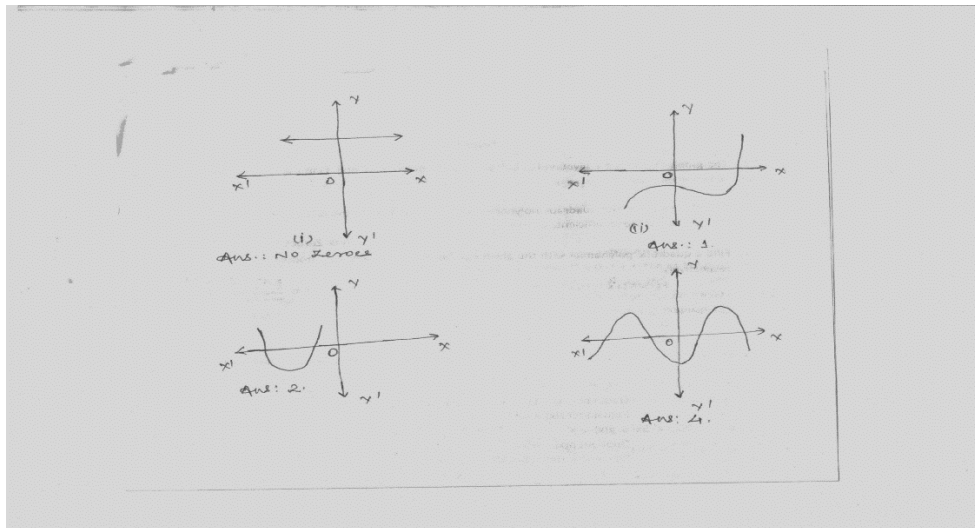
9. If the sum of squares of the zeroes of the polynomials  $6x^2 + x + k$  is  $\frac{25}{36}$ , find the value of k?( CBSE 2014-15)
10. If one zero of the quadratic polynomial  $f(x) = 4x^2 - 8kx - 9$  is negative of the other, then find the value of k?(CBSE 2014-15)
11. Find the values of k for which the quadratic equation  $9x^2 - 3kx + k = 0$  has equal roots. (CBSE 2014)
12. On dividing  $3x^3 - 2x^2 + 5x + 5$  by the polynomial  $p(x)$ , the quotient and remainder are  $x^2 - x + 2$  and  $-7$  respectively. Find  $p(x)$ ? (CBSE 2013)
13. Find all the zeroes of the polynomial  $x^4 + x^3 - 9x^2 - 3x + 18$ , if two of its zeroes are  $\sqrt{3}$  and  $\sqrt{-3}$ . (CBSE 2010,13)
14. If  $\alpha, \beta$  are zeroes of the quadratic polynomial  $p(x) = x^2 - (k - 6)x + (2k + 1)$ . Find the value of k if  $\alpha + \beta = \alpha\beta$ . (CBSE 2010)
15. If the zeroes of the polynomial  $x^2 - 5x + k$  are the reciprocal of each other, then find the value of K? (CBSE 2011)
16. If  $\alpha$  and  $\beta$  are zeroes of the quadratic polynomial  $x^2 - 6x + a$ , find the value of  $a'$ . If  $3\alpha + 2\beta = 20$ . (CBSE 2010)

### LEVEL III

17. On dividing  $3x^3 + 4x^2 + 5x - 13$  by a polynomial  $g(x)$ , the quotient and remainder are  $3x + 10$  and  $16x - 43$  respectively. Find the polynomial  $g(x)$ . (CBSE 14-15)
18. If  $-5$  is a root of quadratic equation  $2x^2 + px - 15 = 0$  and the quadratic equation  $p(x^2 + x)k = 0$  has equal roots, find the value of  $k$ . (CBSE 2106)
19. If  $\alpha, \beta$  and  $\gamma$  are zeroes of the polynomial  $6x^3 + 3x^2 - 5x + 1$ , then find the values of  $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ . (CBSE 2010)
20. Form a cubic polynomial whose zeroes are 3, 2 and  $-1$ . Hence find
- Sum of its zeroes
  - Sum of the product, taken two at a time
  - Product of its zero.

### (SELF EVALUATION QUESTIONS)

21. Find the number of zeroes of  $p(x)$  in each case, for some polynomials  $p(x)$ .



22. If  $\alpha$  and  $\beta$  are the zeroes of the equation  $6x^2 + x - 2 = 0$ , find  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
23. If one of the zeroes of the polynomial  $2x^2 + px + 4 = 0$  is 2, find the other zero, also find the value of  $p$
24. If one zero of the polynomial  $(a^2 + 9)x^2 + 13x + 6a$  is reciprocal of the other. Find the value of  $a$ . (All India)

## **Value Based Questions**

25. If  $\alpha$  be the number of person who take junk food,  $\beta$  be the person who take food at home and  $\alpha$  and  $\beta$  be the zeroes of quadratic polynomial  $f(x) = x^2 - 3x + 2$ , then find a quadratic polynomial whose zeroes are  $\frac{1}{2\alpha+\beta}$  and  $\frac{1}{2\beta+\alpha}$ , which way of taking food you prefer and why?
26. If the number of apples and mangoes are the zeroes of the polynomial  $3x^2 = 8x - 2k + 1$  and the number of apples is 7 times the number of mangoes, then find the number of zeroes and value of  $k$ . What are benefits of fruits in our daily life?

# Pair of Linear Equations in Two Variables

## (Key Points)

- An equation of the form  $ax + by + c = 0$ , where  $a, b, c$  are real nos. ( $a \neq 0, b \neq 0$ ) i.e. ( $a^2 + b^2 \neq 0$ ) is called a linear equation in two variables  $x$  and  $y$ .

Ex : (i)  $x - 5y + 2 = 0$

(ii)  $\frac{3}{2}x - y = 1$

- The general form for a pair of linear equations in two variables  $x$  and  $y$  is

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where  $a_1, b_1, c_1, a_2, b_2, c_2$  are all real nos and  $a_1 \neq 0, b_1 \neq 0, a_2 \neq 0, b_2 \neq 0$ .

Examples:  $x + 3y - 6 = 0$

$$2x - 3y - 12 = 0$$

- Graphical representation of a pair of linear equations in two variables:

$$a_1x + b_1y + c_1 = 0$$

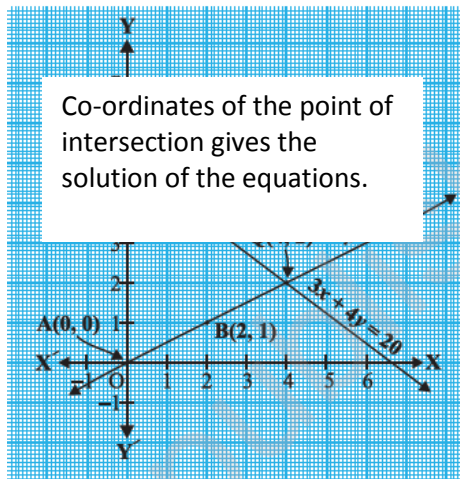
$$a_2x + b_2y + c_2 = 0$$

- (i) Will represent intersecting lines if  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e. unique solution. And these types of equations are called consistent pair of linear equations.

Ex:  $x - 2y = 0$

$$3x + 4y - 20 = 0$$

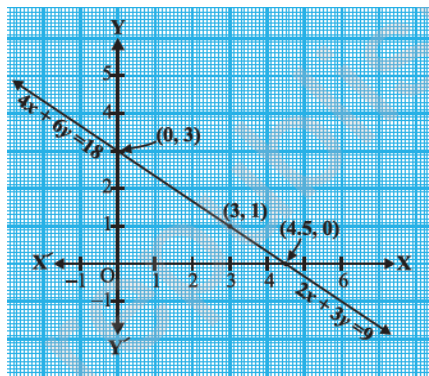


- (ii) will represent overlapping or coincident lines if  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e. Infinitely many solutions, consistent or dependent pair of linear equations

Ex:  $2x + 3y - 9 = 0$

$$4x + 6y - 18 = 0$$



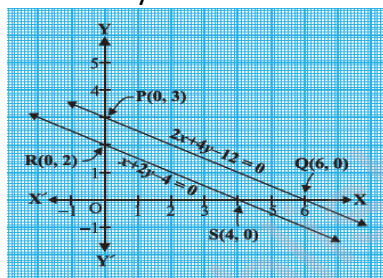
The graph is Coincident lines,

(iii) will represent parallel lines if  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. no solution and called inconsistent pair of linear equations.

Ex:  $x + 2y - 4 = 0$

$2x + 4y - 12 = 0$



Parallel lines, no solution.

- Algebraic methods of solving a pair of linear equations:

- Substitution method
- Elimination Method
- Cross multiplication method

## Level - I

- Find the value of 'a' so that the point(2,9) lies on the line represented by  $ax - 3y = 5$
- Find the value of k so that the lines  $2x - 3y = 9$  and  $kx - 9y = 18$  will be parallel.
- Find the value of k for which  $x + 2y = 5$ ,  $3x + ky + 15 = 0$  is inconsistent
- Check whether given pair of lines is consistent or not  $5x - 1 = 2y$ ,  $y = \frac{-1}{2} + \frac{5}{2}x$
- Determine the value of 'a' if the system of linear equations  $3x + 2y - 4 = 0$  and  $ax - y - 3 = 0$  will represent intersecting lines.
- Write any one equation of the line which is parallel to  $\sqrt{2}x - \sqrt{3}y = 5$
- Find the point of intersection of line  $-3x + 7y = 3$  with x-axis
- For what value of k the following pair has infinite number of solutions.  
 $(k-3)x + 3y = k$   
 $k(x+y) = 12$
- Write the condition so that  $a_1x + b_1y = c_1$  and  $a_2x + b_2y = c_2$  have unique solution.

## Level - II

1. 5 pencils and 7pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.
2. Solve the equations:  
 $3x - y = 3$   
 $7x + 2y = 20$
3. Find the fraction which becomes to  $\frac{2}{3}$  when the numerator is increased by 2 and equal to  $\frac{4}{7}$  when the denominator is increased by 4
4. Solve the equation:  
 $px + qy = p - q$   
 $qx - py = p + q$
5. Solve the equation using the method of substitution:  
 $3x - 5y = -1$   
 $x - y = -1$
6. Solve the equations:  
 $\frac{1}{2x} - \frac{1}{y} = -1$   
 $\frac{1}{x} + \frac{1}{2y} = 8$       Where,  $x \neq 0, y \neq 0$
7. Solve the equations by using the method of cross multiplication:  
 $x + y = 7$   
 $5x + 12y = 7$

## Level - III

1. Draw the graph of the equations  
 $4x - y = 4$   
 $4x + y = 12$   
Determine the vertices of the triangle formed by the lines representing these equations and the x-axis. Shade the triangular region so formed
2. Solve Graphically  
 $x - y = -1$  and  
 $3x + 2y = 12$

Calculate the area bounded by these lines and the x- axis,

3. Solve :- for u & v

$$4u - v = 14uv$$

$$3u + 2v = 16uv \text{ where } u \neq 0, v \neq 0$$

4. Ritu can row downstream 20 km in 2 hr , and upstream 4 km in 2 hr . Find her speed of rowing in still water and the speed of the current. (HOTS)

5. In a  $\triangle ABC$ ,  $\angle C = 3\angle B = 2(\angle A + \angle B)$  find the these angle. (HOTS)

6. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by 1 man alone and that by one boy alone to finish the work. (HOTS)

7. Find the value of K for which the system of linear equations  $2x+5y = 3$ ,  $(k+1)x + 2(k+2)y = 2K$  will have infinite number of solutions. (HOTS)

### **SELF EVALUTION**

1. Solve for x and y:

$$x + y = a + b$$

$$ax - by = a^2 - b^2$$

2. For what value of k will the equation  $x + 5y - 7 = 0$  and  $4x + 20y + k = 0$  represent coincident lines?

3. Solve graphically:  $3x + y + 1 = 0$

$$2x - 3y + 8 = 0$$

4. The sum of digits of a two digit number is 9. If 27 is subtracted from the number, the digits are reversed. Find the number.

5. Draw the graph of  $x + 2y - 7 = 0$  and  $2x - y - 4 = 0$ . Shade the area bounded by these lines and Y-axis.

6. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of the students in the class.

7. A man travels 370 km partly by train and remaining by car. If he covers 250 km by train and the rest by the car it takes him 4 hours, but if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.
8. Given linear equation  $2x + 3y - 8 = 0$ , write another linear equation such that the geometrical representation of the pair so formed is (i) intersecting lines, (ii) Parallel Lines.
9. Solve for x and y.  
 $(a-b)x + (a+b)y = a^2 - 2ab - b^2$   
 $(a+b)(x+y) = a^2 + b^2$  (CBSE 2004, '07C, '08)
10. The sum of two numbers is 8 and the sum of their reciprocal is  $\frac{8}{15}$ . Find the numbers.  
(CBSE 2009)

### Value Based Questions

Q1. The owner of a taxi cab company decides to run all the cars he has on CNG fuel instead of petrol/diesel. The car hire charges in city comprises of fixed charges together with the charge for the distance covered. For a journey of 12km, the charge paid Rs.89 and for a journey of 20 km, the charge paid is Rs. 145.

- What will a person have to pay for travelling a distance of 30 km?
- Which concept has been used to find it?
- Which values of the owner have been depicted here?

Q2. Riya decides to use public transport to cover a distance of 300 km. She travels this distance partly by train and remaining by bus. She takes 4 hours if she travels 60km by bus and the remaining by train. If she travels 100 km by bus and the remaining by train, she takes 10 minutes more.

- Find speed of train and bus separately.
- Which concept has been used to solve the above problem?
- Which values of Riya have been depicted here?



## ANSWER

### LEVEL-I

Q1.  $a = 16$

Q2.  $k = 6$

Q3.  $k = 6$

Q4. Consistent

Q5.  $a \neq \frac{-3}{2}$

Q6.  $5\sqrt{2}x - 5\sqrt{3}y = 5\sqrt{5}$  (May be another solution also)

Q7.  $(-1, 0)$

Q8.  $k = 6$

Q9.  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

### LEVEL-II

Q1.: Cost of one pencil = Rs. 3

Cost of one pen = Rs. 5

Q2.  $x=2, y=3$

Q3.  $28/45$

Q4.  $x = 1, y = -1$

Q5.  $x = -2, y = -1$

Q6.  $x = \frac{1}{6}, y = \frac{1}{4}$

Q7.  $x = 11, y = -4$

### LEVEL-III

Q1.  $(2,4)(1,0)(3,0)$

Q2.  $x = 2, y = 3$  and area =  $7.5 \text{ unit}^2$

Q3.  $u = \frac{1}{2}, v = \frac{1}{4}$

Q4. Speed of the rowing in still water =  $6 \text{ km/hr}$

Speed of the current =  $4 \text{ km/hr}$ .

Q5.  $\angle A = 20^\circ, \angle B = 40^\circ, \angle C = 120^\circ$ .

Q6.: One man can finish work in 140 days.

One boy can finish work in 280 days.

Q7.  $K = 3$

### SELF EVALUATION

Q1.  $X=a, y=b$

Q2.  $K=-28$

Q3.  $X = -1, y = 2$

Q4.  $63$

Q6.  $60$

Q7.Speed of the train=100km/h, speed of the car=80km/h

Q8.(i)  $4x-3y-8=0$  (may be another equation also)

(ii)  $4x+6y+16=0$  (may be another equation also)

Q9. $X= a+b, y = -2ab/(a+b)$

Q10.3,5

## **VALUE BASED QUESTIONS**

Q1.(i)Rs.215,(ii)A pair of linear equations in two variables has been used to find it.

(iii) Awareness of environment.

Q2. (i) The speed of the train = 80 km/h, the speed of the bus = 60km/h

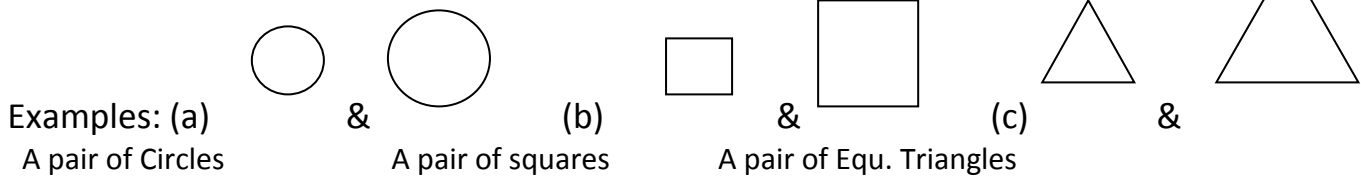
(ii) A pair of linear equations in two variables has been used.

(iii)Controlling the pollution of the environment.

# Triangles

## Key Points

**Similar Figures:** Two figures having similar shapes (size may or may not be same), called Similar figures.



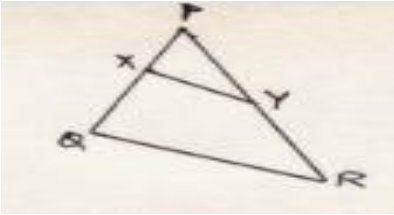
- Pairs of all regular polygons, containing equal number of sides are examples of Similar Figures.
- **Similar Triangles:** Two Triangles are said to be similar if
  - (a) Their corresponding angles are equal (also called Equiangular Triangles)
  - (b) Ratio of their corresponding sides are equal/proportional
- All congruent figures are similar but similar figures may /may not be congruent
- Conditions for similarity of two Triangles
  - (a) AAA criterion/A-A corollary
  - (b) SAS similarity criterion
  - (c) SSS similarity criterion (where 'S' stands for ratio of corresponding sides of two Triangles)

### Important Theorems of the topic Triangles

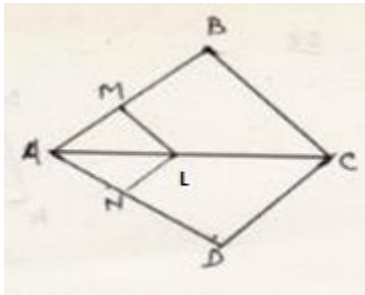
- (a) Basic Proportionality Theorem (B.P.T.)/Thale's Theorem
- (b) Converse of B.P.T.
- (c) Area related theorem of Similar Triangles
- (d) Pythagoras Theorem
- (e) Converse of Pythagoras Theorem

## Level I

- (1) In the figure  $XY \parallel QR$ ,  $PQ/XQ = 7/3$  and  $PR = 6.3\text{cm}$  then find  $YR$



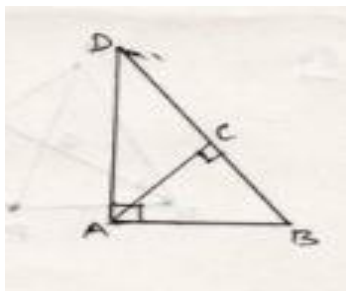
- (2) If  $\triangle ABC \sim \triangle DEF$  and their areas be  $64\text{cm}^2$  &  $121\text{cm}^2$  respectively, then find  $BC$  if  $EF = 15.4\text{ cm}$
- (3)  $ABC$  is an isosceles  $\triangle$ , right angled at  $C$  then prove that  $AB^2 = 2AC^2$
- (4) If  $\triangle ABC \sim \triangle DEF$ ,  $\angle A = 46^\circ$ ,  $\angle E = 62^\circ$  then the measure of  $\angle C = 72^\circ$ . Is it true? Give reason.
- (5) The ratio of the corresponding sides of two similar triangles is  $16:25$  then find the ratio of their perimeters.
- (6) A man goes  $24\text{ km}$  in due east and then He goes  $10\text{ km}$  in due north. How far is He from the starting Point?
- (7) The length of the diagonals of a rhombus is  $16\text{cm}$  &  $12\text{cm}$  respectively then find the perimeter of the rhombus.
- (8) In the figure  $LM \parallel CB$  and  $LN \parallel CD$  then prove that  $AM/AB = AN/AD$



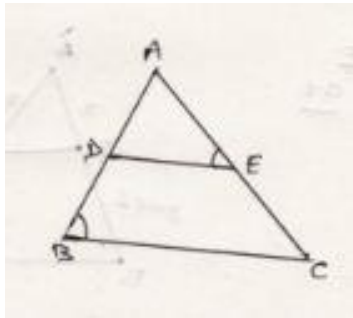
- (9) Which one is the sides of a right angled triangles among the following (a)  $6\text{cm}, 8\text{cm}$  &  $11\text{cm}$  (b)  $3\text{cm}, 4\text{cm}$  &  $6\text{cm}$  (c)  $5\text{cm}$ ,  $12\text{cm}$  &  $13\text{cm}$

## Level II

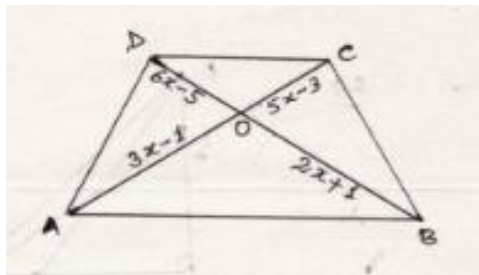
- (1) In the figure  $ABD$  is a triangle right angled at  $A$  and  $AC$  is perpendicular to  $BD$  then show that  $AC^2 = BC \times DC$



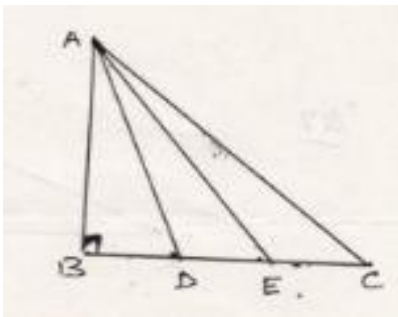
- (2) Two poles of height 10m & 15 m stand vertically on a plane ground. If the distance between their feet is  $5\sqrt{3}$ m then find the distance between their tops.
- (3) D & E are the points on the sides AB & AC of  $\triangle ABC$ , as shown in the figure. If  $\angle B = \angle AED$  then show that  $\triangle ABC \sim \triangle AED$



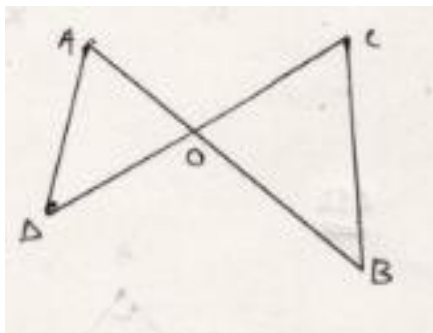
- (4) In the adjoining figure  $AB \parallel DC$  and diagonal AC & BD intersect at point O. If  $AO = (3x-1)$ cm,  $OB = (2x+1)$ cm,  $OC = (5x-3)$ cm and  $OD = (6x-5)$ cm then find the value of x.



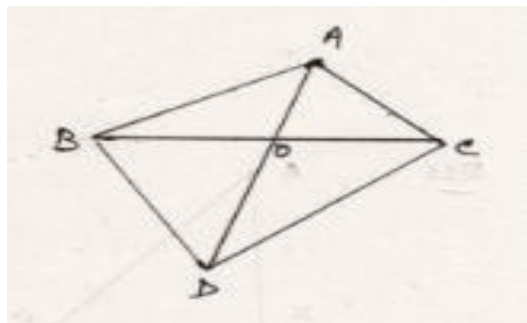
- (5) In the figure D & E trisect BC. Prove that  $8AE^2 = 3AC^2 + 5AD^2$



- (6) In the figure  $OA/OC = OD/OB$  then prove that  $\angle A = \angle C$

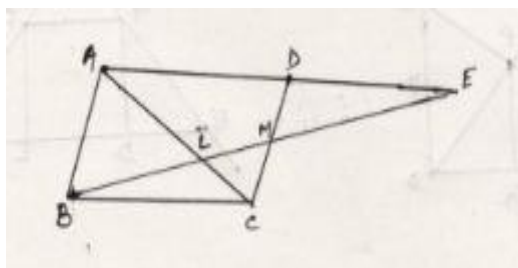


- (7) Using converse of B.P.T. prove that the line joining the mid points of any two sides of a triangle is parallel to the third side of the triangle.
- (8) In the given figure  $\triangle ABC$  &  $\triangle DBC$  are on the same base BC . if AD intersect BC at O then prove that  $\text{ar}(\triangle ABC)/\text{ar}(\triangle DBC) = AO/DO$

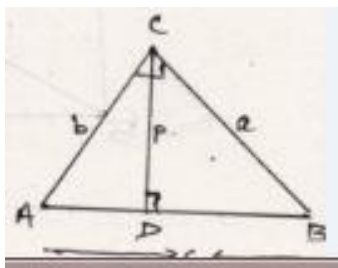


### Level III

- (1) A point O is in the interior of a rectangle ABCD, is joined with each of the vertices A, B, C & D. Prove that  $OA^2 + OC^2 = OB^2 + OD^2$
- (2) In an equilateral triangle ABC, D is a point on the base BC such that  $BD = \frac{1}{3} BC$ , then show that  $9AD^2 = 7AB^2$
- (3) Prove that in a rhombus, sum of squares of the sides is equal to the sum of the squares of its diagonals
- (4) In the adjoining figure ABCD is a parallelogram. Through the midpoint M of the side CD, a line is drawn which cuts diagonal AC at L and AD produced at E. Prove that  $EL = 2BL$

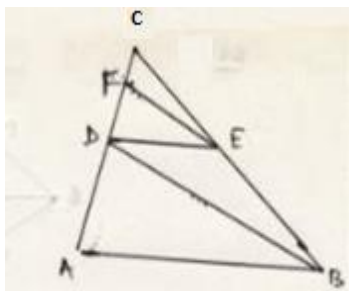


- (5) ABC & DBC are two triangles on the same base BC and on the same side of BC with  $\angle A = \angle D = 90^\circ$ . If CA & BD meet each other at E then show that  $AE \times EC = BE \times ED$
- (6) ABC is a Triangle, right angle at C and p is the length of the perpendicular drawn from C to AB. By expressing the area of the triangle in two ways show that (i)  $pc = ab$  (ii)  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



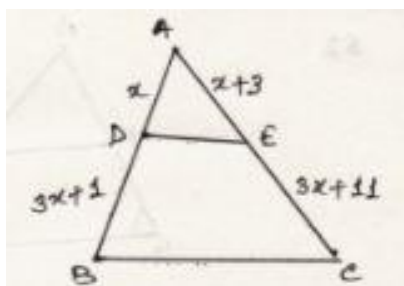
(7) Prove that the ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

(8) In the figure  $AB \parallel DE$  and  $BD \parallel EF$ . Prove that  $DC^2 = CF \times AC$



### Self-Evaluation Questions including Board Questions & Value Based Questions

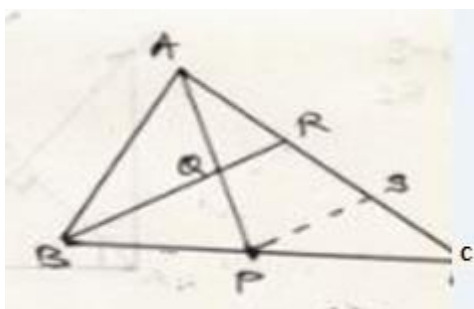
(1) Find the value of  $x$  for which  $DE \parallel BC$  in the adjoining figure



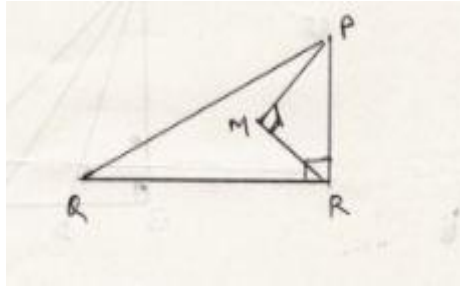
(2) In an equilateral triangle prove that three times the square of one side is equal to four times the square of one of its altitude.

(3) The perpendicular from A on the side BC of a triangle ABC intersect BC at D such that  $DB = 3CD$ . Prove that  $2AB^2 = 2AC^2 + BC^2$

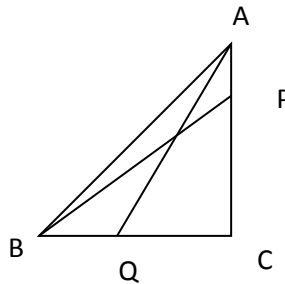
(4) In the adjoining figure P is the midpoint of BC and Q is the midpoint of AP. If BQ when produced meets AC at R, then prove that  $RA = \frac{1}{3} CA$



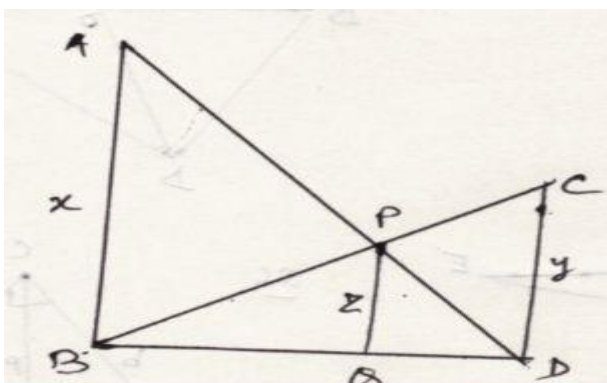
- (5) BL and CM are medians of triangle ABC, right angled at A then prove that  $4(BL^2 + CM^2) = 5BC^2$
- (6) In  $\triangle ABC$  if  $AB = 6\sqrt{3}\text{cm}$ ,  $AC = 12\text{cm}$  and  $BC = 6\text{cm}$  then show that  $\angle B = 90^\circ$
- (7) In the adjoining figure  $\angle QRP = 90^\circ$ ,  $\angle PMR = 90^\circ$ ,  $QR = 26\text{cm}$ ,  $PM = 8\text{cm}$  and  $MR = 6\text{cm}$  then find the area of  $\triangle PQR$



- (8) If the ratio of the corresponding sides of two similar triangles is 2:3 then find the ratio of their corresponding altitudes.
- (9) In the adjoining figure ABC is a  $\triangle$  right angled at C. P & Q are the points on the sides CA & CB respectively which divides these sides in the ratio 2:1, then prove that  $9(AQ^2 + BP^2) = 13 AB^2$



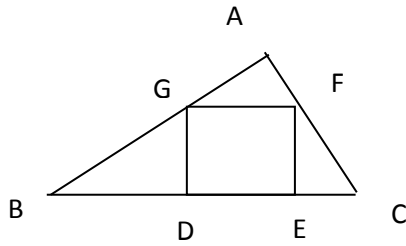
- (10) In the adjoining figure  $AB \parallel PQ \parallel CD$ ,  $AB = x$  unit,  $CD = y$  unit &  $PQ = z$  unit then prove that  $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$



- (11) State and prove Pythagoras theorem. Using this theorem find the distance between the tops of two vertical poles of height 12m & 18m respectively fixed at a distance of 8m apart from each other.



(12) in the adjoining figure DEFG is a square &  $\angle BAC = 90^\circ$  then prove that (a)  $\triangle AGF \sim \triangle DBG$   
 (B)  $\triangle AGF \sim \triangle EFC$  (C)  $\triangle DBG \sim \triangle EFC$  (D)  $DE^2 = BD \times EC$



- (13) A man steadily goes 4 m due east and then 3m due north .Find  
 (a) Distance from initial point to last point.  
 (b) What mathematical concept is used in this problem?  
 (c) What is its value?

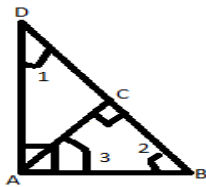
## Solutions

### Level I

- (1) By B.P.T.  $PQ/XQ=PR/YR \Rightarrow 7/3=6.3/YR \Rightarrow YR= 3 \times 6.3/7=2.7$   
 So  $YR=2.7\text{cm}$
- (2) By theorem  $\text{Ar of } \triangle ABC/\text{Ar of } \triangle DEF= BC^2/15.4^2$   
 $\Rightarrow 64/121 = BC^2/15.4^2 \Rightarrow \text{solving } BC = 11.2 \text{ cm}$
- (3) By Pythagoras theorem  $AB^2= AC^2+BC^2 \Rightarrow AB^2= AC^2+AC^2$  (given that  $AC=BC$ )  
 So  $AB^2=2AC^2$
- (4)  $\triangle ABC \sim \triangle DEF \Rightarrow \angle A=\angle D=46^\circ, \angle B=\angle E=62^\circ$  so  $\angle C =180-(46+62)=72^\circ$   
 So it is true.
- (5) Let  $\triangle ABC \sim \triangle DEF$   
 then  $AB/DE= BC/EF=AC/DF= \text{perimeter of } \triangle ABC/\text{Perimeter of } \triangle DEF$   
 $\Rightarrow AB/DE=\text{perimeter of } \triangle ABC/\text{Perimeter of } \triangle DEF$   
 So  $\text{perimeter of } \triangle ABC/\text{Perimeter of } \triangle DEF=16:25$
- (6) By Pythagoras theorem ,  $\text{Distance} = \sqrt{24^2+10^2}$   
 On Solving ,  $\text{distance} =26\text{km}$
- (7) In  $\triangle AOD$ , by Pythagoras theorem  $AD=\sqrt{6^2+8^2}$   
 $\Rightarrow AD= 10\text{cm}$   
 So  $\text{perimeter of Rhombus} = 4 \times 10\text{cm}$   
 $= 40\text{cm}$
- (8) In  $\triangle ABC$ ,  $LM \parallel BC$  so by BPT  $AM/AB=AL/AC$ ----- (i)  
 Similarly in  $\triangle ACD$ ,  $LN \parallel DC$ , so by BPT  $AN/AD = AL/AC$ ----- (ii)  
 Comparing results i & ii we get  $AM/AB= AN/AD$

Using Pythagoras thermo ,finding the value of  $p^2+b^2$  &  $h^2$  separately in each case , it comes equal in case of c where  $p^2+b^2$  comes equal to  $h^2$   
 So sides given in question c is the sides of right triangle

### Level II



- (1) In  $\triangle ABC$   $\angle 2 + \angle 3 = 90^\circ$   
 $\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$   
 $\Rightarrow \angle 1 = \angle 3$   
 $\triangle ACD \sim \triangle BCA$   
 $\Rightarrow AC/BC = CD/AC$   
 So  $AC^2 = BC \times CD$

(2) Using Pythagoras theorem

$$\text{Distance between their tops} = \sqrt{5^2 + (5\sqrt{3})^2}$$

$$\sqrt{25 + 75}$$

Distance between their tops = 10m

(3) In  $\triangle AED$  &  $\triangle ABC$

$$\angle AED = \angle ABC \text{ (given)}$$

$$\angle A = \angle A \text{ (common)}$$

By AA corollary  $\triangle ABC \sim \triangle AED$

(4) Diagonals of a trapezium divide each proportionally

$$\text{So } AO/OC = BO/OD$$

$$3x-1/5x-3 = 2x+1/6x-5$$

$$\Rightarrow 8x^2 - 20x + 8 = 0$$

Solving we get  $x = 2 \text{ \& } 1/2$  (na)

So  $x = 2$

(5)  $BD = DE = EC = P$  (let)

$$BE = 2P \text{ \& } BC = 3P$$

$$\text{In Rt } \triangle ABD, AD^2 = AB^2 + BD^2$$

$$= AB^2 + p^2$$

$$\text{In Rt } \triangle ABE, AE^2 = AB^2 + BE^2$$

$$= AB^2 + (2p)^2$$

$$= AB^2 + 4p^2$$

$$\text{In Rt } \triangle ABC, AC^2 = AB^2 + BC^2$$

$$= AB^2 + (3p)^2$$

$$= AB^2 + 9p^2$$

$$\text{Now taking RHS } 3AC^2 + 5AD^2$$

$$= 3(AB^2 + 9p^2) + 5(AB^2 + p^2)$$

$$= 8AB^2 + 32p^2$$

$$= 8(AB^2 + 4p^2)$$

$$= 8AE^2$$

$$= \text{LHS}$$

(6)  $OA/OC = OD/OB$  (given)

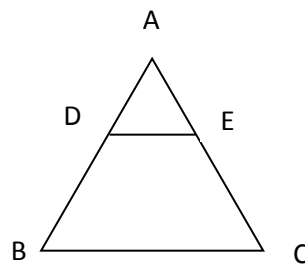
$$\Rightarrow OA/OD = OC/OB$$

$$\text{\& } \angle AOD = \angle BOC \text{ (v.o. } \angle^s)$$

By SAS similarity condition  $\triangle AOD \sim \triangle COB$

$$\Rightarrow \angle A = \angle C$$

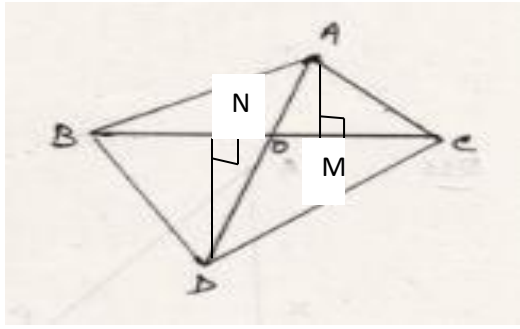
(7)



Given that  $AD/DE = 1$  &  $AE/EC = 1$  (as D & E are mid points of the sides AB & AC)

$$\Rightarrow AD/DB = AE/EC$$

By converse of BPT  $DE \parallel BC$



(8)

We draw perpendiculars AM & DN as shown.  $\triangle DON \sim \triangle AOM$  (by AA corollary)

$$DN/AM = OD/OA \Rightarrow AM/DN = OA/OD \text{-----(i)}$$

$$\begin{aligned} \text{Ar of } \triangle ABC / \text{Ar of } \triangle DBC &= (1/2 \times BC \times AM) / (1/2 \times BC \times DN) \\ &= AM/DN \end{aligned}$$

$$\text{Ar of } \triangle ABC / \text{Ar of } \triangle DBC = AO/OD \text{ (from (i))}$$

### Level III

(1) We draw  $PQ \parallel BC$  through Pt. O  $\Rightarrow BPQC$  &  $APQD$  are rectangles.

$$\text{In Rt } \triangle OPB, \text{ by Pythagoras theorem } OB^2 = BP^2 + OP^2 \text{-----(i)}$$

$$\text{In Rt } \triangle OQD, OD^2 = OQ^2 + DQ^2 \text{-----(ii)}$$

$$\text{In Rt } \triangle OQC, OC^2 = OQ^2 + CQ^2 \text{-----(iii)}$$

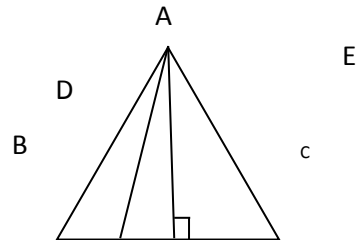
$$\text{In Rt } \triangle OAP, OA^2 = AP^2 + OP^2 \text{-----(iv)}$$

On adding (i) & (ii)

$$\begin{aligned} OB^2 + OD^2 &= BP^2 + OP^2 + OQ^2 + PQ^2 \\ &= CQ^2 + OP^2 + OQ^2 + AP^2 \quad (BP = CQ \text{ \& } DA = AP) \\ &= CQ^2 + OQ^2 + OP^2 + AP^2 \end{aligned}$$

$$\text{So } OB^2 + OD^2 = OC^2 + OD^2$$

(2)



We draw AE perpendicular to BC & AD is joined.

Then  $BD = BC/3$ ,  $DC = 2BC/3$  &  $BE = EC = BC/2$

$$\begin{aligned} \text{In Rt. } \triangle ADE, AD^2 &= AE^2 + DE^2 \\ &= AE^2 + (BE - BD)^2 \\ &= AE^2 + BE^2 + BD^2 - 2 \cdot BE \cdot BD \\ &= AB^2 + (BC/3)^2 - 2 \cdot BC/2 \cdot BC/3 \\ &= AB^2 + BC^2/9 - BC^2/3 \\ &= (9AB^2 + BC^2 - 3BC^2)/9 \\ &= (9AB^2 + AB^2 - 3AB^2)/9 \quad (\text{Given } AB = BC = AC) \\ &= 7AB^2/9 \\ &\Rightarrow 9AD^2 = 7AB^2 \end{aligned}$$

$$(3) \text{In Rt. } \triangle AOB, AB^2 = OA^2 + OB^2$$

$$= (AC/2)^2 + (BD/2)^2$$

$$4AB^2 = AC^2 + BD^2 \text{-----(I)}$$

$$\text{Similarly } 4BC^2 = AC^2 + BD^2 \text{-----(II)}$$

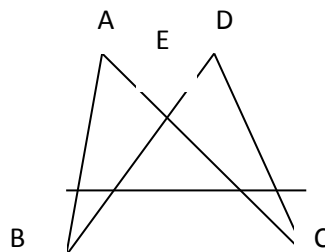
$$4CD^2 = AC^2 + BD^2 \text{-----(III)}$$

$$4AD^2 = AC^2 + BD^2 \text{-----(IV)}$$

$$\text{Adding these results } 4(AB^2 + BC^2 + CD^2 + AD^2) = 4(AC^2 + BD^2)$$

$$\Rightarrow (AB^2 + BC^2 + CD^2 + AD^2) = (AC^2 + BD^2)$$

(4)  $\triangle BMC \cong \triangle EDM$  (by ASA criterion)  
 $\Rightarrow$  by cpct  $DE=BC$  &  $AD=BC$  (opp. sides of  $\parallel$  gm)  
 Adding above results  $AD+DE=BC+BC$   
 $\Rightarrow AE=2BC$   
 Now  $\triangle AEL \sim \triangle CBL$  (By AA corollary)  
 $EL/BL=AE/BC \Rightarrow EL/BL=2BC/BC \Rightarrow EL=2BL$



(5)  $\triangle AEB \sim \triangle DEC$  (AA corollary)  
 $AE/DE=EB/EC$   
 $\Rightarrow AE \times EC=BE \times ED$

(6) Ar of  $\triangle ABC = \frac{1}{2} \times AB \times DC$   
 $= \frac{1}{2} \times c \times p$   
 $= pc/2$   
 Again Ar of  $\triangle ABC = \frac{1}{2} \times AC \times BC$   
 $= \frac{1}{2} \times b \times a$   
 $= ab/2$

Comparing above two areas

$$ab/2 = pc/2$$

$$\Rightarrow ab=pc$$

Now in Rt  $\triangle ABC$ ,  $AB^2=BC^2+AC^2$

$$c^2=a^2+b^2$$

$$(ab/p)^2 = a^2+b^2 \quad (ab=pc \Rightarrow c=ab/p)$$

$$a^2b^2/p^2 = a^2+b^2$$

$$1/p^2 = a^2+b^2/a^2b^2$$

$$1/p^2 = 1/a^2 + 1/b^2$$

(7) Theorem question, as proved

(8) In  $\triangle ABC$ ,  $AB \parallel DE$ , by BPT  $AC/DC = BC/CE$  ----- (i)

In  $\triangle BDC$ ,  $EF \parallel BD$ , by BPT  $DC/CF = BC/EC$  ----- (ii)

Comparing (i) & (ii)  $AC/DC = DC/CF$

$$\Rightarrow DC^2 = AC \times CF$$

## Self-Evaluation Questions

(1) A/Q  $AD/DB = AE/EC$  (by BPT)

$$\Rightarrow x/3x+1 = x+3/3x+11$$

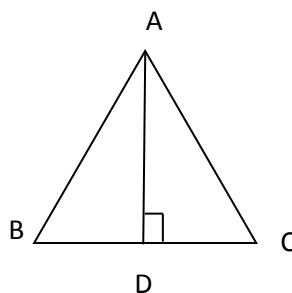
$$\Rightarrow 3x^2+11=3x^2+9x+x+3$$

$$\text{So } x=3$$

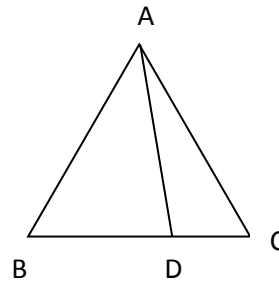
(2)

In  $\triangle ABD$ ,  $AB^2=AD^2+BD^2$

$$= AD^2+(BC/2)^2 \quad (AB=BC=AC)$$

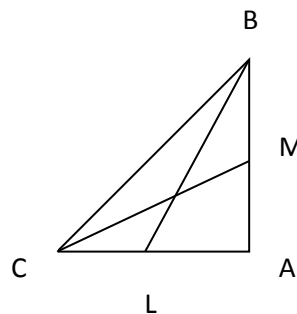


$$\begin{aligned}
 &= AD^2 + AB^2/4 \\
 4AB^2 &= 4AD^2 + AB^2 \\
 4AB^2 - AB^2 &= 4AD^2 \\
 3AB^2 &= 4AD^2
 \end{aligned}$$



$$\begin{aligned}
 (3), BC &= 4CD \Rightarrow CD = BC/4 \\
 \Rightarrow BD &= 3CD = 3BC/4 \quad \text{-----(i)} \\
 \text{In } \triangle ABD, AB^2 &= AD^2 + BD^2 \quad \text{-----(ii)} \\
 \text{In } \triangle ACD, AC^2 &= AD^2 + CD^2 \quad \text{-----(iii)} \\
 \text{Now } AB^2 - AC^2 &= BD^2 - CD^2 \\
 &= 9BC^2/16 - BC^2/16 = BC^2/2 \\
 2(AB^2 - AC^2) &= BC^2 \\
 2AB^2 &= 2AC^2 + BC^2
 \end{aligned}$$

(4) we draw  $PS \parallel BR$   
 In triangle RBC, P is the mid point of BC and  $PS \parallel BR$   
 $RS = CS$  [Mid point theorem] .....(1)  
 In  $\triangle APS$ ,  $PS \parallel BR$  ie  $PS \parallel QR$  and Q is the mid point of AP  
 So  $AR = RS$ .....[II](Mid point theorem)  
 From results (I)&(II)  $AR = RS = CS$   
 So  $AR = 1/3 AC$



$$\begin{aligned}
 (5) \\
 \text{In } \triangle ABL, BL^2 &= AB^2 + AL^2 \\
 4BL^2 &= 4AB^2 + 4AL^2 \\
 &= 4AB^2 + (2AL)^2 \\
 4BL^2 &= 4AB^2 + AC^2 \quad \text{-----(i)} \\
 \text{In } \triangle ACM \\
 4CM^2 &= 4AC^2 + AB^2 \quad \text{-----(ii)} \\
 \text{On adding} \\
 4BL^2 + 4CM^2 &= 4AB^2 + AC^2 + 4AC^2 + AB^2 \\
 &= 5AB^2 + 5AC^2 \\
 &= 5(AC^2 + AB^2) \\
 &= 5BC^2
 \end{aligned}$$

$$\begin{aligned}
 \text{ie } 4BL^2 + 4CM^2 &= 5BC^2 \\
 (6) AC^2 &= 12^2 = 144 \quad \text{-----(i)}
 \end{aligned}$$

$$\begin{aligned}
 AB^2 + BC^2 &= (6\sqrt{3})^2 + 6^2 \\
 &= 108 + 36 \\
 AB^2 + BC^2 &= 144 \quad \text{-----(ii)}
 \end{aligned}$$

From (i)&(ii)  
 $AC^2 = AB^2 + BC^2$  (converse of Pythagoras theorem)  
 $\angle B = 90^\circ$

$$\begin{aligned}
 (7) \text{ In } \triangle PMR \\
 PR^2 &= PM^2 + MR^2 \\
 &= 6^2 + 8^2 \\
 &= 36 + 64
 \end{aligned}$$

$$= 100$$

$$PR = 10\text{cm}$$

$$\text{In } \Delta PQR \text{ } PQ^2 = QR^2 - PR^2$$

$$= 26^2 - 10^2$$

$$= 676 - 100$$

$$= 576$$

$$PQ = 24\text{cm}$$

$$\text{Now Area of } \Delta PQR = \frac{1}{2} \times PR \times PQ$$

$$= \frac{1}{2} \times 10 \times 24$$

$$= 120 \text{ cm}^2$$

8. Ratio of areas of two similar  $\Delta^s$  is equal to the ratio of squares of corresponding sides

$$\text{So Ratio of areas of two similar } \Delta^s = (2x/3x)^2 = 4/9$$

So Ratio of areas of two similar  $\Delta^s$  = ratio of squares of their corresponding altitudes =  $4/9$

So, Ratio of corresponding altitudes =  $4/9$

9. P divide CA in the ratio 2 :1 Therefore

$$CP = \frac{2}{3} AC \dots\dots\dots(i)$$

$$QC = \frac{2}{3} BC \dots\dots\dots(ii)$$

In Right Triangle ACQ:

$$AQ^2 = QC^2 + AC^2$$

$$\text{Or, } AQ^2 = \frac{4}{9} BC^2 + AC^2 \text{ (QC = } \frac{2}{3} BC \text{)}$$

$$\text{Or, } 9 AQ^2 = 4 BC^2 + 9 AC^2 \dots\dots\dots(iii)$$

Similarly, In Right Triangle BCP

$$9BP^2 = 9BC^2 + 4 AC^2 \dots\dots\dots(iv)$$

Adding eq. (iii) & (iv)

$$9(AQ^2 + BQ^2) = 13(BC^2 + AC^2)$$

$$9(AQ^2 + BQ^2) = 13AB^2$$

10. In triangle ABD,

PQ || AB

$$PQ/AB = DQ/BD$$

$$\text{Or, } Z/X = DQ/BD \dots\dots\dots(i)$$

In triangle BCD,

PQ || CD

$$PQ/CD = BQ/BD$$

$$\text{Or, } Z/Y = BQ/BD \dots\dots\dots(ii)$$

Adding eq. (i) & (ii)

$$Z/X + Z/Y = DQ/BD + BQ/BD = DQ + BQ/BD$$

$$\text{Or, } Z/X + Z/Y = BD/BD = 1$$

$$\text{Or, } 1/X + 1/Y = 1/Z$$

11. State and Prove Pythagoras Theorem

$$AP = AB - PB = (18 - 12) \text{ m} = 6 \text{ m}$$

$$[ PB = CD = 6 \text{ m} ]$$

$$PC = BD = 8 \text{ m}$$

In  $\Delta ACP$

$$AC = \sqrt{AP^2 + PC^2}$$

$$= \sqrt{(6)^2 + (8)^2}$$

$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

$$AC = 10 \text{ m}$$

(12) DE // GF & AC cuts them

$\Rightarrow \angle DAG = \angle FGC$  (corres.  $\angle^s$ )  
 $\angle GDE = 90^\circ \Rightarrow \angle GDA = 90^\circ$   
 $\triangle ADG \sim \triangle GCF$  (By AA corollary, shown above)  
(ii) similarly  $\triangle FEB \sim \triangle GCF$   
Since  $\triangle ADG$  &  $\triangle FEB$  are both similar to  $\triangle GCF$   
 $\Rightarrow \triangle ADG \sim \triangle FEB$   
(iii)  $\triangle ADG \sim \triangle FEB$   
 $AD/FE = DG/FB$   
 $\Rightarrow AD/DG = EF/EB$   
(iv)  $\triangle ADG \sim \triangle FEB$   
 $AD/FE = DG/FB$   
 $\Rightarrow AD/DE = DE/EB$  ( $FE = DG = DE$ )  
 $DE^2 = AD \times EB$

(13)(i) distance from the initial point  $= \sqrt{3^2 + 4^2}$   
 $= \sqrt{25}$   
 $= 5\text{m}$

(ii) Pythagoras theorem  
(iii) To save time & energy



# INTRODUCTION TO TRIGONOMETRY

## IMPORTANT CONCEPTS (TAKE A LOOK)

**1. TRIGONOMETRY**---A branch of mathematics in which we study the relationships between the sides and angles of a triangle, is called trigonometry.

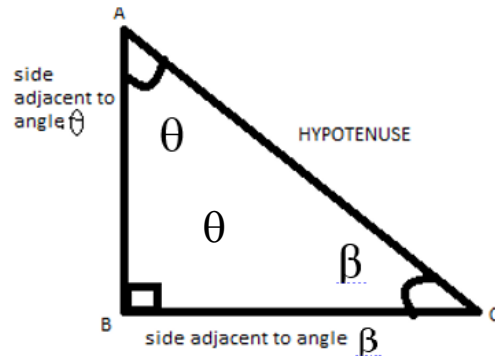
**2. TRIGONOMETRIC RATIOS** -----Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.

Trigonometric ratios of an acute angle in a right angled triangle ---

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB}$$



$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta} = \frac{AC}{AB}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to } \angle \theta} = \frac{AC}{BC}$$

For  $\angle \beta$ ,  $\sin \beta = AB/AC$ ,  $\cos \beta = BC/AC$ ,  $\tan \beta = AB/BC$   
 $\operatorname{Cosec} \beta = AC/AB$ ,  $\sec \beta = AC/BC$ ,  $\cot \beta = BC/AB$

3. Relationship between different trigonometric ratios

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

4. Trigonometric Identity---- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

Important trigonometric identities:

- (i)  $\sin^2\theta + \cos^2\theta = 1$
- (ii)  $1 + \tan^2\theta = \sec^2\theta$
- (iii)  $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

5. Trigonometric Ratios of some specific angles.

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric ratios of complementary angles.

- (i)  $\sin (90^\circ - \theta) = \cos \theta$
- (ii)  $\cos (90^\circ - \theta) = \sin \theta$
- (iii)  $\tan (90^\circ - \theta) = \cot \theta$
- (iv)  $\cot (90^\circ - \theta) = \tan \theta$
- (v)  $\sec (90^\circ - \theta) = \operatorname{cosec} \theta$
- (vi)  $\operatorname{cosec} (90^\circ - \theta) = \sec \theta$

## Level – I

1. If  $\theta$  and  $3\theta-30^\circ$  are acute angles such that  $\sin\theta=\cos(3\theta-30^\circ)$ , then find the value of  $\tan\theta$ .

2. Find the value of  $\frac{(\cos 30^\circ + \sin 60^\circ)}{(1 + \cos 60^\circ + \sin 30^\circ)}$

3. Find the value of  $(\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2$

4. If  $\tan\theta = \frac{3}{4}$  then find the value of  $\cos^2\theta - \sin^2\theta$

5. If  $\sec\theta + \tan\theta = p$ , then find the value of  $\sec\theta - \tan\theta$

6. Change  $\sec^4\theta - \sec^2\theta$  in terms of  $\tan\theta$ .

7. Prove that  $\frac{\sin^3\alpha + \cos^3\alpha}{\sin\alpha + \cos\alpha}$

$$+ \sin\alpha \cos\alpha = 1$$

(CBSE 2009)

8. In a triangle ABC, it is given that  $\angle C = 90^\circ$  and  $\tan A = 1/\sqrt{3}$ , find the value of  $(\sin A \cos B + \cos A \sin B)$

(CBSE 2008)

9. Find the value of  $\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ$ .

10. If  $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$ , then find the value of  $x$

11. If  $0^\circ \leq x \leq 90^\circ$  and  $2\sin^2 x = 1/2$ , then find the value of  $x$

12. Find the value of  $\operatorname{cosec}^2 30^\circ - \sin^2 45^\circ - \sec^2 60^\circ$

13. Simplify  $(\sec\theta + \tan\theta)(1 - \sin\theta)$

14. Prove that  $\cos A / (1 - \sin A) + \cos A / (1 + \sin A) = 2 \sec A$

## Level – II

1. If  $\sec\alpha = 5/4$  then evaluate  $\tan\alpha / (1 + \tan^2\alpha)$ .

2. If  $A+B = 90^\circ$ , then prove that  $\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$

3. If  $7 \sin^2 A + 3 \cos^2 A = 4$ , show that  $\tan A = 1/\sqrt{3}$ .

(CBSE 2008)

4. Prove that  $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

5. Prove that  $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$ . (CBSE 2008, 2009C)

6. Evaluate  $\frac{11 \sin 70^\circ}{7 \cos 20^\circ} - \frac{4 \cos 53^\circ \operatorname{cosec} 37^\circ}{7 \tan 15^\circ \tan 35^\circ \tan 55^\circ \tan 75^\circ}$

7. Find the value of  $\sin 30^\circ$  geometrically.

8. If  $\tan (A - B) = \sqrt{3}$ , and  $\sin A = 1$ , then find A and B.

9. If  $\theta$  is an acute angle and  $\sin \theta = \cos \theta$ , find the value of  $3 \tan^2 \theta + 2 \sin^2 \theta - 1$ .

10. If  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$  and  $\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$ , prove that  $x^2/a^2 + y^2/b^2 = 2$ .

11. Prove that  $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$ .

### Level - III

1. Evaluate the following:  $-\sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)$

2. If  $\frac{\cos \alpha}{\cos \beta} = m$ , and  $\frac{\cos \alpha}{\sin \beta} = n$ , show that  $(m^2 + n^2) \cos^2 \beta = n^2$ . (CBSE 2012)

3. Prove that  $\tan^2 \theta + \cot^2 \theta + 2 = \operatorname{cosec}^2 \theta \sec^2 \theta$

4. If  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ , then show that  $(\cos \theta - \sin \theta) = \sqrt{2} \sin \theta$ .

(CBSE 1997, 2002, 2007)

5. Prove that  $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$ .

6. Prove that  $\sin \theta / (1 - \cos \theta) + \tan \theta / (1 + \cos \theta) = \sec \theta \operatorname{cosec} \theta + \cot \theta$ .

7. If  $x = a \sin \theta$  and  $y = b \tan \theta$ . Prove that  $a^2/x^2 - b^2/y^2 = 1$ .

8. Prove that  $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$ .

9. Prove that  $(\sec \theta + \tan \theta - 1) / (\tan \theta - \sec \theta + 1) = \cos \theta / (1 - \sin \theta)$ .

10. Prove that  $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$  (CBSE 2005, 07, 08)

11. Evaluate  $\frac{\sin^2\theta + \sin^2(90^\circ - \theta)}{3(\sec^2 61^\circ - \cot^2 29^\circ)} - \frac{3\cot^2 30^\circ \sin^2 54^\circ \sec^2 36^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$

12. If  $\sin\theta + \cos\theta = m$  and  $\sec\theta + \operatorname{cosec}\theta = n$ , then prove that  $n(m^2 - 1) = 2m$ .

### Self-Evaluation

1. If  $a \cos\theta + b \sin\theta = c$ , then prove that  $a \sin\theta - b \cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$ .

2. If A, B, C are interior angles of triangle ABC, show that  $\operatorname{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2 \frac{A}{2} = 1$ .

3. If  $\sin\theta + \sin^2\theta + \sin^3\theta = 1$ , prove that  $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$ .

4. If  $\tan A = n \tan B$ ,  $\sin A = m \sin B$ , prove that  $\cos^2 A = (m^2 - 1)/(n^2 - 1)$ .

5. Evaluate:  $\frac{\sec\theta \operatorname{cosec}(90^\circ - \theta) - \tan\theta \cot(90^\circ - \theta) + \sin^2 55^\circ + \sin^2 35^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}$

6. If  $\sec\theta + \tan\theta = p$ , prove that  $\sin\theta = (p^2 - 1)/(p^2 + 1)$ .

7. Prove that  $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$ .

8. Prove that:  $\frac{\cos\theta}{1 - \tan\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$

9. Prove that  $\frac{1 + \cos A + \sin A}{1 + \cos A - \sin A} = \frac{1 + \sin A}{\cos A}$ .

10. Prove that  $(1 + \cos\theta + \sin\theta) / (1 + \cos\theta - \sin\theta) = (1 + \sin\theta) / \cos\theta$

# STATISTICS

(i) Assumed Mean method or Shortcut method

$$\text{Mean} = \bar{X} = a + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$$

Where a = assumed mean

And  $d_i = X_i - a$

(ii) Step deviation method.

$$\text{Mean} = \bar{X} = a + \frac{\sum_{i=1}^n f_i u_i}{\sum_{i=1}^n f_i} \times h$$

Where a = assumed mean

h = class size

And  $u_i = (X_i - a)/h$

- Median of a grouped frequency distribution can be calculated by

$$\text{Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$$

Where

l = lower limit of median class

n = number of observations

cf = cumulative frequency of class preceding the median class

f = frequency of median class

h = class size of the median class.

- Mode of grouped data can be calculated by the following formula.

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Where

l = lower limit of modal class

h = size of class interval

f<sub>1</sub> = Frequency of the modal class

f<sub>0</sub> = frequency of class preceding the modal class

f<sub>2</sub> = frequency of class succeeding the modal class

- Empirical relationship between the three measures of central tendency.

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

$$\text{Or, Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

- Ogive

Ogive is the graphical representation of the cumulative frequency distribution. It is of two types:

(i) Less than type ogive.

(ii) More than type ogive

- Median by graphical method

The x-coordinated of the point of intersection of 'less than ogive' and 'more than ogive' gives the median.

### LEVEL – I

Slno	Question																
1	What is the mean of 1 <sup>st</sup> ten prime numbers?																
2	What measure of central tendency is represented by the abscissa of the point where less than ogive and more than ogive intersect?																
3	If the mode of a data is 45 and mean is 27, then median is _____.																
4	Find the mode of the following <table><tr><td>X<sub>i</sub></td><td>35</td><td>38</td><td>40</td><td>42</td><td>44</td><td></td></tr><tr><td>f<sub>i</sub></td><td>5</td><td>9</td><td>10</td><td>7</td><td>2</td><td></td></tr></table>	X <sub>i</sub>	35	38	40	42	44		f <sub>i</sub>	5	9	10	7	2			
X <sub>i</sub>	35	38	40	42	44												
f <sub>i</sub>	5	9	10	7	2												
5	Write the median class of the following distribution. <table><tr><td>Class</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr><tr><td>Frequency</td><td>4</td><td>4</td><td>8</td><td>10</td><td>12</td><td>8</td><td>4</td></tr></table>	Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Frequency	4	4	8	10	12	8	4
Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70										
Frequency	4	4	8	10	12	8	4										
6	The wickets taken by a bowler in 10 cricket matches are as follows: 2, 6 ,4 ,5, 0, 2, 1, 3, 2, 3 Find the mode of the data																
7.	How one can find median of a frequency distribution graphically																
8.	What important information one can get by the abscissa of the point of intersection of the less than type and the more than type commulative frequency curve of a group data																

### LEVEL – II

Slno	Question	Ans							
1	Find the median of the following frequency distribution	167							
	Height in cm		160-162	163-165	166-168	169-171	172-174		
	Frequency		15	117	136	118	14		
2	Given below is the distribution of IQ of the 100 students. Find the median IQ	106.1							
	IQ		75-84	85-94	95-104	105-114	115-124	125-134	135-144
	Frequency		8	11	26	31	18	4	2
3	Find the median of the following distribution	28.5							
	Class interval		0-10	10-20	20-30	30-40	40-50	50-60	
	Frequency		5	8	20	15	7	5	
4	A class teacher has the following absentee record of 40 students of a class for the whole								

	term.																	
	<table><tr><td>No. of days</td><td>0-6</td><td>6-10</td><td>10-14</td><td>14-20</td><td>20-28</td><td>28-38</td><td>38-40</td></tr><tr><td>No. of students</td><td>11</td><td>10</td><td>7</td><td>4</td><td>4</td><td>3</td><td>1</td></tr></table> <p>Write the above distribution as less than type cumulative frequency distribution.</p>	No. of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40	No. of students	11	10	7	4	4	3	1	
No. of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40											
No. of students	11	10	7	4	4	3	1											
5	<p>Using the assumed mean method find the mean of the following data.</p> <table><tr><td>Class interval</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td></td><td></td></tr><tr><td>frequency</td><td>7</td><td>8</td><td>12</td><td>13</td><td>10</td><td></td><td></td></tr></table>	Class interval	0-10	10-20	20-30	30-40	40-50			frequency	7	8	12	13	10			Ans 27.2
Class interval	0-10	10-20	20-30	30-40	40-50													
frequency	7	8	12	13	10													
6	Name the keyword used for central tendency	Mean , median , mode																

### LEVEL – III

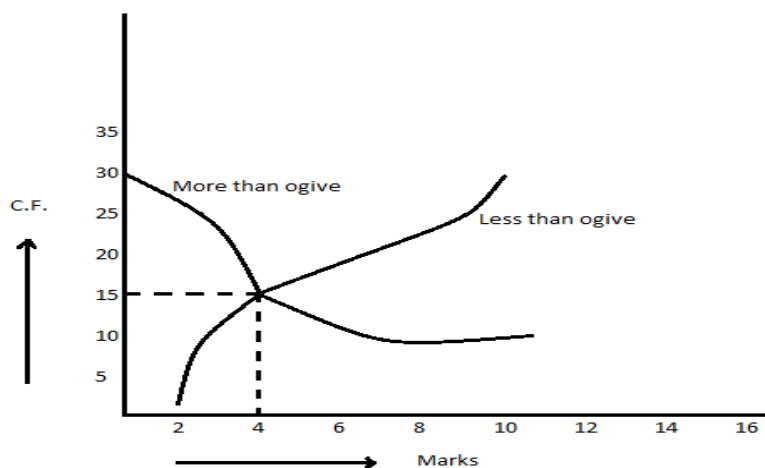
SN	Question						Ans			
1	If the mean distribution is 25						P=16			
	Class	0-10	10-20	20-30	30-40	40-50				
	Frequency	5	18	15	P	6				
	Then find p.									
2	Find the mean of the following frequency distribution using step deviation method						25			
	Class	0-10	10-20	20-30	30-40	40-50				
	Frequency	7	12	13	10	8				
3	Find the value of p if the median of the following frequency distribution is 50						P=10			
	Class	20-30	30-40	40-50	50-60	60-70		70-80	80-90	
	Frequency	25	15	P	6	24		12	8	
4	Find the median of the following data						76.36			
	Marks	Less Than 10	Less Than 30	Less Than 50	Less Than 70	Less Than 90		Less Than 110	Less Than 130	Less than 150
	Frequency	0	10	25	43	65		87	96	100
5	Compare the modal ages of two groups of students appearing for entrance examination.									
	Age in yrs	16-18	18-20	20-22	22-24	24-26				
	Group A	50	78	46	28	23				



	Group B	54	89	40	25	17		
6	The mean of the following frequency distribution is 57.6 and the sum of the observations is 50. Find the missing frequencies $f_1$ and $f_2$ .							$f_1 = 8$ and $f_2 = 10$
	Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
	Frequency	7	$f_1$	12	$f_2$	8	5	50
7	The following distribution give the daily income of 65 workers of a factory							
	Daily income (in Rs)	100-120	120-140	140-160	160-180	180-200		
	No. of workers	14	16	10	16	9		
	Convert the above to a more than type cumulative frequency distribution and draw its ogive.							
8	Draw a less than type and more than type ogives for the following distribution on the same graph. Also find the median from the graph.							
	Marks	30-39	40-49	50-59	60-69	70-79	80-89	90-99
	No. of students	14	6	10	20	30	8	12

### SELF – EVALUATION

- What is the value of the median of the data using the graph in figure of less than ogive and more than ogive?



- If mean = 60 and median = 50, then find mode using empirical relationship.
- Find the value of  $p$ , if the mean of the following distribution is 18.

Variate ( $x_i$ )	13	15	17	19	$20+p$	23
Frequency ( $f_i$ )	8	2	3	4	$5p$	6

- Find the mean, mode and median for the following data.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	5	8	15	20	14	8	5

5. The median of the following data is 52.5. find the value of x and y, if the total frequency is 100.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	2	5	X	12	17	20	Y	9	7	4

6. Draw 'less than ogive' and 'more than ogive' for the following distribution and hence find its median.

Classes	20-30	30-40	40-50	50-60	60-70	70-80	80-90
frequency	10	8	12	24	6	25	15

7. Find the mean marks for the following data.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
No. of students	5	9	17	29	45	60	70	78	83	85

8. The following table shows age distribution of persons in a particular region. Calculate the median age.

Age in years	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80
No. of persons	200	500	900	1200	1400	1500	1550	1560

9. If the median of the following data is 32.5. Find the value of x and y.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
frequency	x	5	9	12	y	3	2	40

10. The following are ages of 300 patients getting medical treatment in a hospital on a particular day.

Age( in years)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of patients	60	42	55	70	53	20

Draw:

1. Less than type cumulative frequency distribution
2. More than type cumulative frequency distribution

### Value Based Question

Q1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality.

Monthly consumption (in units)	65 – 85	85 – 105	105 – 125	125- 145	145- 165	165 – 185	185 – 205
Number of consumers	4	5	13	20	14	8	4

Mr. Sharma always saves electricity by switching of all the electrical equipment just immediately after their uses. So , his family belongs to the group 65- 85 .

- (i) Find the median of the above data
- (ii) How many families consumed 125 or more units of electricity during a month?
- (iii) What moral values of Mr. Sharma have been depicted in this situation?

Q2. The mileage (km per litre) of 50 cars of the same models is tested by manufacturers and details are tabulated as given below:-

Mileage (km per litre)	10 – 12	12 – 14	14 - 16	16- 18
No. of cars	7	12	18	13

- i. Find the mean mileage.
- ii. The manufacturer claims that the mileage of the model is 16km/litre. Do you agree with this claim?
- iii. Which values do you think the manufacturer should imbibe in his life?

## ANSWER

1. 12.9
2. MEDIAN
3. 33
4. MODE = 40
5. MEDIAN = 30-40
6. 2
7. OGIVE
8. Median

## Level II

Q1 167

Q2 106.1

Q3 28.5l

### Q4

<u>No. of days</u>	<u>Less Than 6</u>	<u>Less Than 10</u>	<u>Less Than 14</u>	<u>Less Than 20</u>	<u>Less Than 28</u>	<u>Less Than 38</u>	<u>Less Than 40</u>
<u>No. of students</u>	<u>11</u>	<u>21</u>	<u>28</u>	<u>32</u>	<u>36</u>	<u>39</u>	<u>40</u>

### Q5 27.2

### Q6 Mean, median, mode

## **MODEL SAMPLE PAPER – SA 1**

### **BLUE PRINT**

#### **CLASS-X**

#### **Sub:-Mathematics**

S. N.	Name of Chapter	VSA	SA-I	SA-II	LA	Total
1	Number system	2(2)	1(2)	1(3)	1(4)	5(11)
2	Algebra	1(1)	2(4)	2(6)	3(12)	8(23)
3	Geometry	1(1)	1(2)	2(6)	2(8)	6(17)
4	Trigonometry	-----	1(2)	4(12)	2(8)	7(22)
5	Statistics	-----	1(2)	1(3)	3(12)	5(17)
	Total	4(4)	6(12)	10(30)	11(44)	31(90)

Note: - Number of questions is given outside the brackets and marks are given within the bracket.

## MODEL SAMPLE PAPER – SA1

Time Allowed: - 3 hours

Max. Marks:-90

General instruction:-

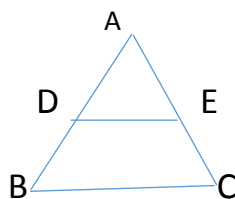
- Question should be distributed to the students before 15 minutes of the commencement of examination.
- All questions are compulsory.
- The questions paper comprises of 31 questions divided into four sections A, B, C and D. you are to attempt all the four sections.
- Question no. 1 to 4 in section 'A' is of 1 mark each.  
Question no. 5 to 10 in section 'B' are of 2 marks each.  
Question no. 11 to 20 in section 'C' are of 3 marks each.  
Question no. 21 to 31 in section 'D' are of 4 marks each.
- Use of calculator is not permitted.

सामान्य निर्देश

- प्रश्न पत्र छात्रों को परीक्षा शुरू होने के 15 मिनट पहले वितरित करना है।
- सभी प्रश्न अनिवार्य हैं।
- इस प्रश्न पत्र में कुल 31 प्रश्न हैं जिन्हें चार खण्डों में बांटा गया है।
- खंड में 1 से 4 प्रश्न हैं। प्रत्येक प्रश्न 1 अंक का है।  
खंड में 5 से 10 प्रश्न हैं। प्रत्येक प्रश्न 2 अंक का है।  
खंड में 11 से 20 प्रश्न हैं। प्रत्येक प्रश्न 3 अंक का है।  
खंड में 21 से 31 प्रश्न हैं। प्रत्येक प्रश्न 4 अंक का है।
- केलकुलेटर का उपयोग वर्जित है।

### Section: - A

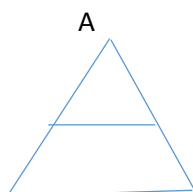
- If the HCF of 55 and 99 is expressible in the form of  $55m - 99$  then find the value of  $m$ .  
यदि 55 और 99 का HCF को  $55m - 99$  के रूप व्यक्त किया गया हो तो  $m$  का मान निकाले।
- If  $\frac{241}{4000} = \frac{241}{2^m \times 5^n}$ , find the values of  $m$  and  $n$  where  $m$  &  $n$  are whole number.  
यदि  $\frac{241}{4000} = \frac{241}{2^m \times 5^n}$ , तो  $m$  और  $n$  का मान ज्ञात करें जहाँ  $m$  और  $n$  पूर्ण संख्याएँ हैं।
- For what value of  $K$ ,  $(-4)$  is a zero of the polynomial  $x^2 - x - (2k + 2)$ ?  
“ $K$ ” के किस मान के लिए  $-4$ , बहुपद  $x^2 - x - (2k + 2)$  का शून्यांक होगा?
- In  $\triangle ABC$  shown in figure  $DE \parallel BC$ . If  $BC = 8\text{cm}$ ,  $DE = 6\text{cm}$  and area of  $\triangle ADE = 45\text{cm}^2$ , what is the area of  $\triangle ABC$ .



दिये गये चित्र के अनुसार  $\triangle ABC$  में  $DE \parallel BC$ . यदि  $BC = 8\text{cm}$ ,  $DE = 6\text{cm}$  और  $\triangle ADE$  का क्षेत्रफल  $= 45\text{cm}^2$  तो  $\triangle ABC$  का क्षेत्रफल क्या होगा?

### Section:-B

- Find a quadratic polynomial whose zero are  $-2$  and  $3$ .  
एक द्विघात बहुपद ज्ञात करें जिसका शून्यांक  $-2$  और  $3$  है।
- Check whether  $6^n$  can end with the digit zero for any natural number  $n$ .  
जाँच करें कि अंक  $6^n$  शून्य के साथ समाप्त हो सकता है या नहीं, जहाँ  $n$  एक प्राकृत संख्या है।
- The larger of two supplementary angles exceeds smaller by  $20^\circ$ . Find the angles.  
दो संपूरक कोणों में से सबसे बड़ा कोण, छोटे कोण से  $20^\circ$  बड़ा है। तो कोणों को ज्ञात करें।
- In the given fig  $DE \parallel BC$ , if  $BD = x - 3$ ,  $AB = 2x$ ,  $CE = x - 2$  and  $AC = 2x + 3$ . Find  $x$ .



D                      E  
B                      C

दिये गये चित्र में  $DE \parallel BC$ , यदि  $BD=x-3$ ,  $AB=2x$ ,  $CE=x-2$  और  $AC=2x+3$ . तो  $x$  का मान निकाले।

9. If  $\cos\theta = \frac{x}{y}$  then find the value of  $\tan\theta$  &  $\sec\theta$ .

यदि  $\cos\theta = \frac{x}{y}$  तो  $\tan\theta$  और  $\sec\theta$  का मान निकाले।

10. If the mean of the following data is 15, find P.

X	5	10	15	20	25
F	6	P	6	10	5

निम्नलिखित सारणी का माध्य यदि 15 है तो P का मान निकाले।

X	5	10	15	20	25
F	6	P	6	10	5

**Section: - C**

11. Prove that  $\sqrt{7}$  is irrational number.

सिद्ध करे कि  $\sqrt{7}$  एक अपरिमेय संख्या है।

12. For what value of P will the following pair of linear equations have infinitely many solutions?

$$(P-3)x + 3y = p; p x + p y = 12$$

निम्नलिखित युग्म रैखिक समीकरणों में P के किस मान के लिए अनंत हल होगा?

$$(P-3)x + 3y = p; p x + p y = 12$$

13. Solve for x & y:

$$\frac{x+1}{2} + \frac{y-1}{3} = 8; \frac{x-1}{3} + \frac{y+1}{2} = 9$$

X और y का हल निकाले।

$$\frac{x+1}{2} + \frac{y-1}{3} = 8; \frac{x-1}{3} + \frac{y+1}{2} = 9$$

14. D and E are points on the side CA and CB respectively of  $\triangle ABC$  right angled at C. prove that

$$AE^2 + BD^2 = AB^2 + DE^2$$

समकोण त्रिभुज ABC जो C पर समकोण है। D और E क्रमशः भुजाएँ CA और CB के बिन्दु हैं तो सिद्ध करे कि

$$AE^2 + BD^2 = AB^2 + DE^2.$$

15. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

सिद्ध करे कि दो समकोण त्रिभुजों के क्षेत्रफल का अनुपात उसके तदनुरूपी माध्यिकाओं के वर्ग के अनुपात के बराबर होती है।

16. Evaluate:-  $\frac{5\sin^2 30^\circ + \cos^2 45^\circ + 4\tan^2 60^\circ}{2\sin 30^\circ \cdot \cos 60^\circ + \tan 45^\circ}$

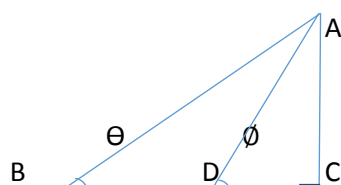
मान निकाले।

$$\frac{5\sin^2 30^\circ + \cos^2 45^\circ + 4\tan^2 60^\circ}{2\sin 30^\circ \cdot \cos 60^\circ + \tan 45^\circ}$$

17. If  $\sin(A+B)=1$  and  $\cos(A-B)=\frac{\sqrt{3}}{2}$ , find A and B.

यदि  $\sin(A+B)=1$  और  $\cos(A-B)=\frac{\sqrt{3}}{2}$ , तो A और B का मान निकालें।

18. In the figure,  $\triangle ABC$  is a right angled triangle, D is the mid-point of BC.  $\angle C=90^\circ$ . Then find  $\frac{\tan\theta}{\tan\phi}$ .



दिये गये चित्र में  $\triangle ABC$  एक समकोण त्रिभुज है। D, BC का मध्य बिन्दु है। कोण  $\angle C=90^\circ$  तो  $\frac{\tan\theta}{\tan\phi}$  का मान निकालें।

19. Prove that  $(1 + \cot A - \operatorname{cosec} A) (1 + \tan A + \sec A) = 2$ .

सिद्ध करें कि  $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ .

20. The mean of the following frequency table is 50. Find the value of x & y.

Class interval	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	X	32	Y	19	120

निम्नलिखित बंटन सारणी में माध्य 50 है, तो X और Y का मान ज्ञात करें।

Class interval	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	X	32	Y	19	120

#### Section: - D

21. Use Euclid's division lemma to show that the cube of any positive integer is of the form  $9m$ ,  $9m + 1$  or  $9m + 8$ .

युक्लिड डिविजन लेंमा का प्रयोग करते हुए दिखाएँ कि किसी भी धनात्मक संख्याओं का घन  $9m$ ,  $9m + 1$  अथवा  $9m + 8$  के रूप में होता है।

22. If two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 + \sqrt{3}$  &  $2 - \sqrt{3}$ . Find other Zeros.

यदि बहुपद  $x^4 - 6x^3 - 26x^2 + 138x - 35$  के दो शून्यांक  $2 + \sqrt{3}$  &  $2 - \sqrt{3}$  हैं। तो दूसरे शून्यांकों को ज्ञात करें।

23. Solve for x & y :

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

X और Y का हल निकालें।

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

24. A boat goes 30 km upstream and 44 km downstream in 10 hrs. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

एक नाव 10 घंटों में 30 किलोमीटर धारा के विपरीत और 44 कि० मी० धारा की दिशा में जाती है। 13 घंटों में 40 कि० मी० धारा के विपरीत और 55 कि० मी० धारा की दिशा में जा सकती है। स्थिर जल में नाव और धारा का वेग ज्ञात करें।

25. State and prove Pythagoras theorem.

पाइथागोरस प्रमेय को लिखें और सिद्ध करें।

26. In an equilateral triangle ABC, D is a point on side BC such that  $BD = \frac{1}{3}BC$ . prove that  $9AD^2 = 7AB^2$

किसी समबाहु त्रिभुज ABC में D, BC पर एक बिन्दु इस तरह है कि  $BD = \frac{1}{3}BC$ . सिद्ध करें कि  $9AD^2 = 7AB^2$

27. If  $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ , prove that  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ .

यदि  $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ , तो सिद्ध करें कि  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ .

28. Prove that

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cdot \operatorname{cosec} \theta$$

सिद्ध करें कि।

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cdot \operatorname{cosec} \theta$$

29. Calculate the arithmetic mean of the following frequency distribution using the step deviation method.

Class Interval	0-50	50-100	100-150	150-200	200-250	250-300
Frequency	17	35	43	40	21	24

निम्नलिखित बंटन सारणी में माध्य पद विचलन विधि से ज्ञात करें।

Class Interval	0-50	50-100	100-150	150-200	200-250	250-300
----------------	------	--------	---------	---------	---------	---------



Frequency	17	35	43	40	21	24
-----------	----	----	----	----	----	----

30. To highlight child Labour problem, some students organized a javelin through competition. 50 students participated in this completion. The distance (in meters) thrown are recorded below.

Distain (in m)	0-20	20-40	40-60	60-80	80-100
Number of students	6	11	17	12	04

- Construct a cumulative frequency table.
- Draw cumulative frequencies curve (Less than type) and calculate the median distance thrown.
- Which value is depicted by students?

बाल मजदुरी की समस्या को उजागर करने के लिए, कुछ विद्यार्थी जावलीन थ्रो प्रतियोगिता के लिए संगठित हुए। इस प्रतियोगिता में 50 विद्यार्थियों ने भाग लिया। फेंकी गयी दूरियों का रिकार्ड निम्नलिखित है।

दूरी (मिटर में)	0-20	20-40	40-60	60-80	80-100
विद्यार्थियों की संख्या	6	11	17	12	04

- उपर लिखित सारणी की सहायता से संचयी बारंबारता सारणी बनावें।
- संचयी बारंबारता वक्र खींचें और फेंकी गयी दूरियों की माधिका की गणना करें।
- विद्यार्थी द्वारा कौन –सा मूल्य दर्शाता है ?

31. Compare the modal a ages of two groups of students A and B appearing for an entrance test.

Age (in Year)	Group:-A	Group:-B
16-18	50	54
18-20	78	89
20-22	46	40
22-24	28	25
24-26	23	17

विद्यार्थी के दो समूह A और B जो एक प्रतियोगिता परीक्षा में सम्मिलित होते हैं, के बहुलक उम्र की तुलना करें।

उम्र (वर्ष में)	समूह :- A	समूह :-B
16-18	50	54
18-20	78	89
20-22	46	40
22-24	28	25
24-26	23	17

# MODEL SAMPLE PAPER

## MARKING SCHEME

### CLASS –X

#### SA-1

- |  |   |
|--|---|
| 1. $m = 2$                               | 1 |
| 2. $m = 5, n = 3$                        | 1 |
| 3. $K = 9$                               | 1 |
| 4. $\text{ar}\Delta ABC = 80\text{cm}^2$ | 1 |

#### Section-B

- | 5. $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$<br>$= x^2 - (-2 + 3)x + (-6)$<br>$p(x) = x^2 - x - 6$   | 1<br>$\frac{1}{2}$<br>$\frac{1}{2}$             |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
|---|---|---------|------|---|---|----|----|---|-----|----|---|----|----|----|-----|----|---|-----|--|------|---------|--|
| 6. $6^n = 2^n \times 3^n$<br>$\therefore 6^n$ has no 5 as factor.<br>$\therefore 6^n$ can not end with the digit zero.  | 1<br>$\frac{1}{2}$<br>$\frac{1}{2}$             |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 7. Let smaller angle $= x^\circ$<br>$\therefore$ Larger angle $= (x + 20)^\circ$<br>$x + x + 20 = 180^\circ$<br>$\Rightarrow x = 80^\circ$<br>$\therefore$ Smaller angle $= 80^\circ$<br>Larger angle $= 100^\circ$   | 1<br>$\frac{1}{2}$<br><br><br><br>$\frac{1}{2}$ |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 8. $\therefore DE \parallel BC$<br>$\Rightarrow \frac{BD}{AB} = \frac{CE}{AC}$<br>$\Rightarrow \frac{x-3}{2x} = \frac{x-2}{2x+3}$<br>$\Rightarrow x = 9$  | 1<br>$\frac{1}{2}$<br><br>$\frac{1}{2}$         |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 9. $\cos\theta = \frac{x}{y}$<br>$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$<br>$= \frac{\sqrt{1-\frac{x^2}{y^2}}}{\frac{x}{y}} = \frac{\sqrt{y^2-x^2}}{x}$<br>$\sec\theta = \frac{1}{\cos\theta} = \frac{y}{x}$  | 1<br><br><br>$\frac{1}{2}$<br>$\frac{1}{2}$     |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 10.   |   |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <th>X</th> <th>f</th> <th>f(x)</th> </tr> <tr> <td>5</td> <td>6</td> <td>30</td> </tr> <tr> <td>10</td> <td>P</td> <td>10p</td> </tr> <tr> <td>15</td> <td>6</td> <td>90</td> </tr> <tr> <td>20</td> <td>10</td> <td>200</td> </tr> <tr> <td>25</td> <td>5</td> <td>125</td> </tr> <tr> <td></td> <td>27+p</td> <td>445+10p</td> </tr> </table> | X   | f       | f(x) | 5 | 6 | 30 | 10 | P | 10p | 15 | 6 | 90 | 20 | 10 | 200 | 25 | 5 | 125 |  | 27+p | 445+10p |  |
| X   | f   | f(x)    |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 5   | 6   | 30      |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 10  | P   | 10p     |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 15  | 6   | 90      |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 20  | 10  | 200     |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 25  | 5   | 125     |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
|   | 27+p  | 445+10p |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| $x = \frac{445+10p}{27+p}$  | 1   |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| $15 = \frac{445+10p}{27+p}$   | $\frac{1}{2}$                                   |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| $\Rightarrow P = 8$   | $\frac{1}{2}$                                   |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |
| 11. Let $\sqrt{7}$ is a rational no.  | $\frac{1}{2}$                                   |         |      |   |   |    |    |   |     |    |   |    |    |    |     |    |   |     |  |      |         |  |

$$\therefore \sqrt{7} = \frac{p}{q}$$

(p & q are integers, q ≠ 0, having no common factor)  $\frac{1}{2}$

Squaring

$$7 = \frac{p^2}{q^2}$$

$$\Rightarrow p^2 = 7q^2 \text{----- (i)}$$

1

$\therefore p^2$  & p are divisible by 7

let p = 7m

$$p^2 = 49m^2 \text{----- (ii)}$$

from (i) and (ii)

$$q^2 = 7m^2$$

$\Rightarrow q^2$  & q are divisible by 7

$\therefore p$  & q have 7 as common factor.

1

$\therefore$  our supposition is wrong

$\therefore \sqrt{7}$  is irrational number.

$\frac{1}{2}$

12.  $a_1 = p-3, \quad b_1 = 3, \quad c_1 = p$

$a_2 = p, \quad b_2 = p, \quad c_2 = 12$

$\frac{1}{2}$

For infinitely many solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

1

$$\Rightarrow \frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$$

From the 1<sup>st</sup> and 2<sup>nd</sup>

1

$$p=0, p=6$$

from last two

$$p=6, -6$$

$$\Rightarrow p=6 \text{ Ans.}$$

$\frac{1}{2}$

13.  $\frac{x+1}{2} + \frac{y-1}{3} = 8$

$$\Rightarrow 3x + 2y = 47 \text{----- (i)}$$

1

$$\frac{x-1}{3} + \frac{y+1}{2} = 9$$

$$\Rightarrow 2x + 3y = 53 \text{----- (ii)}$$

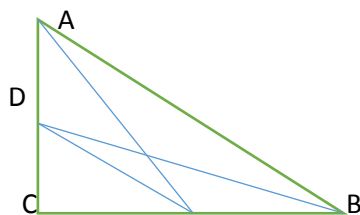
1

Solving (i) & (ii)

$$x=7, y=13$$

1

14.



In right  $\triangle AEC$ ,

$$AE^2 = AC^2 + CE^2 \text{----- (i)}$$

1

In right  $\triangle DBC$

$$BD^2 = CD^2 + BC^2 \text{----- (ii)}$$

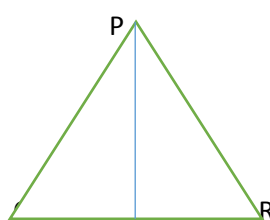
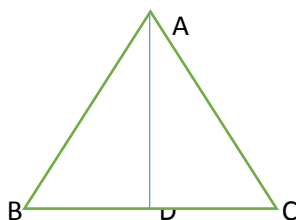
Adding (i) & (ii)

$$AE^2 + BD^2 = AC^2 + CE^2 + CD^2 + BC^2$$

$\frac{1}{2}$

$$= AB^2 + DE^2 \quad \text{Proved}$$

15.



$\frac{1}{2}$   
1

$$\Delta ABC \approx \Delta PQR$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{BD}{QM}$$

In  $\Delta ABC$  &  $\Delta PQM$

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ \& } \angle B = \angle Q \text{ (given)}$$

$$\Rightarrow \Delta ABD \approx \Delta PQM$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

$$\Rightarrow \frac{ar\Delta ABC}{ar\Delta PQR} = \frac{AD^2}{PM^2}$$

$\frac{1}{2}$

$$16. \frac{5 \times \sin^2 30^\circ + \cos^2 45^\circ + 4 \tan^2 60^\circ}{2 \sin 30^\circ \cdot \cos 60^\circ + \tan 45^\circ}$$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times (\sqrt{3})^2}{2 \times \frac{1}{2} \times \frac{1}{2} + 1}$$

2

$$= \frac{\frac{5}{4} + \frac{1}{2} + 12}{\frac{1}{2} + 1} = \frac{\frac{55}{4}}{\frac{3}{2}}$$

$\frac{1}{2}$

$$= \frac{55}{6} \text{ Ans}$$

$\frac{1}{2}$

$$17. \sin(A + B) = \sin 90^\circ$$

$$\Rightarrow A + B = 90^\circ \text{----- (i)}$$

1

$$\cos(A - B) = \cos 60^\circ$$

$$\Rightarrow A - B = 60^\circ$$

1

$$\Rightarrow A = 75^\circ$$

$$\& B = 15^\circ$$

1

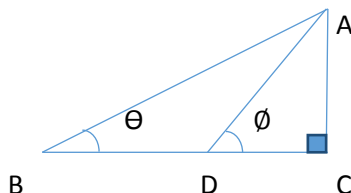
18.

D is the midpoint of BC

$$\therefore BD = DC$$

$$\frac{\tan \theta}{\tan \phi} = \frac{\frac{AC}{BC}}{\frac{AC}{DC}} = \frac{DC}{BC}$$

$$= \frac{DC}{2DC} = \frac{1}{2}$$



1

1

1

$$19. \text{LHS} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

1

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cdot \cos A}$$

1

$$= \frac{\sin^2 A + \cos^2 A + 2 \cos A \cdot \sin A - 1}{\sin A \cdot \cos A}$$

$\frac{1}{2}$

$$= \frac{1 + 2 \cos A \cdot \sin A - 1}{\sin A \cdot \cos A}$$

$$= 2 = \text{RHS.}$$

$\frac{1}{2}$

20.

C.I	Fi	Xi	ui	Fiui
0-20	17	10	-2	-34
20-40	X	30	-1	-x
40-60	32	50 = 9	0	0
60-80	Y	70	1	Y
80-100	19	90	2	38
	$\Sigma f_i = 120$			$\Sigma f_i u_i = 4 - x + y$

$$68 + x + y = 120$$

$$x + y = 52 \text{----- (i)}$$

$X = a + h \frac{\sum f_i u_i}{\sum f_i}$	$\frac{1}{2}$
$50 = 50 = 20 \times \frac{4-x+y}{120} \dots\dots\dots (ii)$	$\frac{1}{2}$
Solving (i) & (ii)	
$X = 28$	
$Y = 24$	$\frac{1}{2}$
21. Let $a = 3q + r, 0 \leq r < 3$	
$\Rightarrow r = 0, 1, 2$	
Let $r = 0$	
$\therefore a = 3q$	
<i>cubing both side</i>	
$a^3 = 27q^3$	
$9 \times (3q^3)$	
$= 9 \times m.$	1
Let. $r = 1$	
$a = 3q + 1$	
$a^3 = (3q + 1)^3$ (Cubing)	
$a^3 = 27q^3 + 3 \times (3q)^2 \times 1 + 3 \times 3q \times 1 + 1^3$	
$= 27q^3 + 27q^2 + 9q + 1$	
$= 9(3q^3 + 3q^2 + q) + 1$	
$= 9m + 1$	1
Let. $r = 2$	
$a = 3q + 2$	
$a^3 = (3q + 2)^3$ (Cubing)	
$a^3 = 27q^3 + 3 \times (3q)^2 \times 2 + 3 \times 3q \times 2^2 + 2^3$	$\frac{1}{2}$
$= 27q^3 + 54q^2 + 36q + 8$	
$= 9(3q^3 + 6q^2 + 4q) + 8$	
$= 9m + 8$	
Cube of any positive integer is of the form $\frac{1}{2}qm, qm+1$ or $qm+8$	
22. $P(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$	
$(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = x^2 - 4x + 1$ is a factor of $p(x)$	1
Now $\frac{x^4 - 6x^3 - 26x^2 + 138x - 35}{x^2 - 4x + 1} = x^2 - 2x - 35$	2
$x^2 - 2x - 35 = x^2 - 7x - 5x - 35$	
$= (x-7)(x-5)$	
$\therefore$ Other zeroes are 7 & -5.	1
23. Let $\frac{1}{3x+y} = a, \frac{1}{3x-y} = b$	
$a + b = \frac{3}{4}$ ----- (i)	1
$\frac{a}{2} - \frac{b}{2} = -\frac{1}{8}$	
$\Rightarrow a - b = -\frac{1}{4}$ ----- (ii)	1
From (i) & (ii)	
$a = \frac{1}{4}, b = \frac{1}{2}$	1
$\Rightarrow 3x + y = 4$	
$3x - y = 2$	
On solving $x = 1, y = 1$	1
24. Let speed of boat in still water = $X$ km/h.	
Speed of stream = $y$ Km/h.	
Speed of boat in downstream = $(x + y)$ km/h.	
Speed of boat in upstream = $(x - y)$ km/h.	1
$\frac{44}{x+y} + \frac{30}{x-y} = 10$	

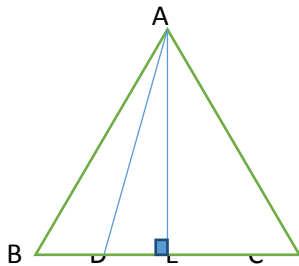
$$\frac{55}{x+y} + \frac{40}{x-y} = 13 \quad 2$$

on solving  $x = 8\text{km/h}$ ,  $y = 3\text{km/h}$  1

25. Fig, given to prove, constructions. 2

Proof 2

26.



Const: - Draw  $AE \perp BC$ .

$\frac{1}{2}$

Proof:-  $\angle BEC = 90^\circ$ ,  $BE = CE (\because AE \text{ is median also})$ .

$$BD = \frac{1}{3}BC.$$

$\triangle ADE$ ,

$$AD^2 = AE^2 + DE^2$$

$$= AB^2 - BE^2 + (BE - BD)^2$$

$$= AB^2 + BD^2 - 2BE \cdot BD$$

$$= AB^2 + \left(\frac{1}{3}BC\right)^2 - BC \times \frac{BC}{3}$$

$$AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} [\because AB = BC = AC]$$

$$\Rightarrow 9AD^2 = 9AB^2 + AB^2 - 3AB^2$$

$$\Rightarrow 9AD^2 = 7AB^2$$

Proved

27.  $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ .

Squaring both sides

$$\cos^2\theta + \sin^2\theta - 2\sin\theta \cdot \cos\theta = 2\sin^2\theta$$

$$\Rightarrow 1 - 2\sin\theta \cdot \cos\theta = 2 - 2\cos^2\theta$$

$$\Rightarrow 1 + 2\sin\theta \cdot \cos\theta = 2\cos^2\theta$$

$$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

28. LHS.  $= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$$= \tan\theta + 1 + \frac{1}{\tan\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + 1 + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta}$$

$$= \frac{1 + \sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta}$$

$$= 1 + \sec\theta \cdot \csc\theta$$

29.

C.I	Fi	Xi	ui	Fiui
0-50	17	25	-2	-34
50-100	35	75	-1	-35
100-150	43	125	0	0
150-200	40	175	1	40
200-250	21	225	2	42

250-300	24	275	3	72
	$\sum f_i = 180$			$\sum f_i u_i = 83$

$$a = 125, h = 50$$

$$x = a + h \frac{\sum f_i u_i}{\sum f_i}$$

$$= 125 + 50 \times \frac{83}{180}$$

$$= 148.06 \text{ Ans.}$$

30. (a) For correct constructing cumulative frequently table.

(b) For drawing correct less than type graph.

(c) Preventing the child labour.

31.

C.I	Group A	Group B
16-18	50	54
18-20	78	89
20-22	46	40
22-24	28	25
24-26	23	17

For Group A:

Modal Class- 18-20

$$l = 18, h = 2, f_1 = 78, f_0 = 50, f_2 = 46$$

$$\text{Modal age for Group A} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 18 + \left( \frac{78 - 50}{156 - 50 - 46} \right) \times 2$$

$$= 18 + \frac{28 \times 2}{60}$$

$$= 18.93$$

For group B:-

$$l = 18, h = 2, f_1 = 89, f_0 = 54, f_2 = 40$$

$$\text{Modal age of Group B} = 18 + \left( \frac{89 - 50}{178 - 54 - 40} \right) \times 2$$

$$= 18.93$$

# ACTIVITES (TERM-I)

## (Any Eight)

- Activity 1: To find the HCF of two Numbers Experimentally Based on Euclid Division Lemma
- Activity 2: To Draw the Graph of a Quadratic Polynomial and observe:
- The shape of the curve when the coefficient of  $x^2$  is positive
  - The shape of the curve when the coefficient of  $x^2$  is negative
  - Its number of zero
- Activity 3: To obtain the zero of a linear Polynomial Geometrically
- Activity 4: To obtain the condition for consistency of system of linear Equations in two variables
- Activity 5: To Draw a System of Similar Squares, Using two intersecting Strips with nails
- Activity 6: To Draw a System of similar Triangles Using Y shaped Strips with nails
- Activity 7: To verify Basic proportionality theorem using parallel line board
- Activity 8: To verify the theorem: Ratio of the Areas of Two Similar Triangles is Equal to the Ratio of the Squares of their corresponding sides through paper cutting.
- Activity 9: To verify Pythagoras Theorem by paper cutting, paper folding and adjusting (Arranging)
- Activity 10: Verify that two figures (objects) having the same shape (and not necessarily the same size) are similar figures. Extend the similarity criterion to Triangles.
- Activity 11: To find the Average Height (in cm) of students studying in a school.
- Activity 12: To Draw a cumulative frequency curve (or an ogive) of less than type.
- Activity 13: To Draw a cumulative frequency curve (or an ogive) of more than type.



## COURSE STRUCTURE (SA-II)

S.NO	TOPIC	MARKS
<b>SA- II</b>		
1	<b>ALGEBRA (CONTD.) QUADRATIC EQUATIONS, ARITHMETIC PROGRESSIONS</b>	23
2	<b>GEOMETRY(CONTD.) CIRCLES, CONSTRUCTIONS</b>	17
3	<b>MENSURATION AREAS RELATED TO CIRCLES, SURFACE AREA &amp; VOLUMES</b>	23
4	<b>TRIGONOMETRY(CONTD.) HEIGHT &amp; DISTANCE</b>	8
5	<b>CO-ORDINATE GEOMETRY</b>	11
6	<b>PROBABILITY</b>	8
<b>TOTAL</b>		90

### TOPIC WISE ANALYSIS OF EXAMPLES AND QUESTIONS NCERT TEXT BOOK

CHAPTERS	TOPICS	Number of Questions for revision		TOTAL
		Questions from solved examples	Questions from exercise	
1	QUADRATIC EQUATIONS	18	24	42
2	ARITHMETIC PROGRESSIONS	16	44	60
3	CO-ORDINATE GEOMETRY	15	25	40
4	SOME APPLICATIONS OF TRIGONOMETRY	7	16	23
5	CIRCLES	3	17	20
6	CONSTRUCTIONS	2	14	16
7	AREA RELATED TO CIRCLES	6	35	41
8	SURFACE AREA & VOLUMES	14	31	45
9	PROBABILITY	13	25	38
Total		94	231	325

**DETAILS OF THE CONCEPTS TO BE MASTERED BY EVERY CHILD OF CLASS X WITH  
EXERCISE AND EXAMPLES OF NCERT TEXT BOOKS.**

**SA - II**

**SYMBOLS USED**

TG/LG is idea identified by term wise error analysis of answers of Q.P. of SA of last three year .

\* - Important Question

\*\* -Very Important Question

\*\*\* -Very Very Important Question

a - Low

b - Average

c - Higher

T.G-Teaching Gap

L.G-Learning Gap

S.No.	TOPIC	CONCEPT	DEGREE OF IMPORTANCE	DIFFICULTY LEVEL		NCERT BOOK
				T.G/L.G	DEGREE OF DIFFICULTY	
01	Quadratic Equation	Standard form of quadratic equation	*	L.G	a	NCERT Text book Q.1.2, Ex 4.1
		Solution of quadratic equation by factorization	***	L.G	a	Example 3,4,5, Q.1, 5 Ex. 4.2
		Solution of quadratic equation by completing the square	**	L.G	b	Example 8,9 Q.1 Ex. 4.3
		Solution of quadratic equation by quadratic formula	***	L.G	a	Example. 10,11,13,14,15 , Q2,3(ii) Ex.4.3
		Nature of roots	***	L.G	a	Example 16 Q.1.2, Ex. 4.4
02	Arithmetic progression	General form of an A.P.	*	L.G	a	Exp-1,2, Ex. 5.1 Q.s2(a), 3(a),4(v)
		nth term of an A.P.	***	L.G	a	Exp. 3,7,8 Ex. 5.2 Q.4,7,11,16,17,18
		Sum of first n terms of an A.P.	** * ** ***	L.G	b	Exp.11,13,15 Ex. 5.3, Q.No.1(i, ii) Q3(i,iii) Q.7,10,12,11,6, Ex5.4, Q-1
03	Coordinate geometry	Distance formula	**	L.G	b	Exercise 7.1, Q.No 1,2,3,4,7,8
		Section formula Midpoint formula	**  ***	L.G	b	Example No. 6,7,9 Exercise 7.2, Q.No. 1,2,4,5 Example 10. Ex.7.2, 6,8,9. Q.No.7
		Area of Triangle	** ***	L.G	a	Ex.1,2,14 Ex 7.3 QNo-12,4 Ex.7.4, Qno-2
04	Some application of Trigonometry	Heights and distances	**	L.G	b	Example-2,3,4 Ex 9.1 Q 2,5,10,12,13,14,15,16
05	Circles	Tangents to a circle	***	L.G	a	Q3(Ex10.1) Q 1,Q6,Q7(Ex 10.2),4

		Number of tangents from a point to a circle	***	L.G	a	Theorem 10.1,10.2 Eg 2.1 Q8,9,,10,12,13(Ex 10.2)
06	Constructions	Division of line segment in the given ratio	*	L.G	b	Const 11.1 Ex 11.1 Qno 1
		Construction of triangle similar to given triangle as per given scale	***	L.G	b	Ex 11.1 Qno-2,4,5,7
		Construction of tangents to a circle	***	L.G	a	Ex 11.2 Qno 1,4
07	Area related to circles	Circumference of a circle	*	L.G	a	Example 1 Exercise 12.1 Q.No 1,2,4
		Area of a circle	*	L.G	a	Example 5,3
		Length of an arc of a circle	*	L.G	a	Exercise 12.2 Q No 5
		Area of sector of a circle	**	L.G	b	Example 2 Exercise 12.2 QNo 1.2
		Area of segment of a circle	**	L.G	a	Exercise 12.2 Qno 4,7,9,3
		Combination of figures	***	L.G	b	Ex 12.3 Example 4.5 1,4,6,7,9,12,15
08	Surface area and volumes	Surface area of a combination of solids	**	T.G	c	Example 1,2,3 Exercise 13.1 Q1,3,6,7,8
		Volume of combination of a solid	**	L.G	b	Example 6 Exercise 13.2 Q 1,2,5,6
		Conversion of solids from one shape to another	***	L.G	a	Example 8 & 10 Exercise 13.3 Q 1,2,6,4,5
		Frustum of a cone	***	L.G	b	Example 12& 14 Exercise 13.4 Q 1,3,4,5 Ex-13.5, Q. 5
09	Probability	Events	*	L.G	a	Ex 15.1 Q4,8,9
		Probability lies between 0 and1	**	L.G	b	Exp- 1,2,4,6,13
		Performing experiment	***	L.G	a	Ex 15 1,13,15,18,24

# QUADRATIC EQUATIONS

## KEY POINTS

1. The general form of a quadratic equation is  $ax^2+bx+c=0$ ,  $a \neq 0$ .  $a$ ,  $b$  and  $c$  are real numbers.
2. A real number  $\alpha$  is said to be a root of quadratic equation  $ax^2 + bx + c = 0$  where  $a \neq 0$  if  $a\alpha^2 + b\alpha + c = 0$ . The zeroes of the quadratic polynomial  $ax^2 + bx + c$  and the roots of the corresponding quadratic equation  $ax^2 + bx + c = 0$  are the same.
3. Discriminant: - The expression  $b^2-4ac$  is called discriminant of the equation  $ax^2+bx+c=0$  and is usually denoted by  $D$ . Thus discriminant  $D=b^2-4ac$ .
4. Every quadratic equation has two roots which may be real, coincident or imaginary.

5. If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2+bx+c=0$  then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{And} \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

6. Sum of the roots,  $\alpha + \beta = -\frac{b}{a}$  and product of the roots,  $\alpha\beta = \frac{c}{a}$

7. Forming quadratic equation, when the roots  $\alpha$  and  $\beta$  are given.

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

8. Nature of roots of  $ax^2+bx+c=0$

- i. If  $D > 0$ , then roots are real and unequal.
- ii.  $D=0$ , then the equation has equal and real roots.
- iii.  $D < 0$ , then the equation has no real roots
- iv. If  $D > 0$  and  $D$  is a perfect square, then roots are rational and unequal.
- v. If  $D > 0$  and  $D$  is not a perfect square then roots are irrational.

9. Irrational roots always occur in conjugate pairs. If  $2+\sqrt{3}$  is one of the root of the quadratic equation then other root is  $2-\sqrt{3}$ .

10. If  $a.b > 0$  then  $a > 0$  and  $b > 0$  or  $a < 0$  and  $b < 0$   
If  $a.b < 0$  then  $a > 0$  and  $b < 0$  or  $a < 0$  and  $b > 0$ .

## LEVEL-I

1. If  $\frac{1}{2}$  is a root of the equation  $x^2+kx-5/4=0$ , then find the value of  $K$ .
2. If  $D > 0$ , then write the roots of a quadratic equation  $ax^2+bx+c=0$
3. Find the Discriminant of  $x^2+5x+5=0$ .
4. Find the the sum of roots of a quadratic equation  $x^2+4x-32=0$
5. Find the product of roots of a quadratic equation  $2x^2+7x-4=0$ .
6. Find the values of  $K$  for which the equation  $9x^2+2kx+1=0$  has real roots.
7. Find the Value of  $K$  if the equation  $x^2-2(k+1)x+k^2=0$  has equal roots.
8. For what value of  $k$ ,  $x=a$  is a solution of equation  $x^2-(a+b)x+k=0$  ?

9. Represent the situation in the form of Quadratic equation:

The Product of Rahman's age (in years) 5 years ago with his age 9 years later is 15.

10. Find the roots of  $x^2 - 3x - 10 = 0$

11. The product of two consecutive odd numbers is 483. Find the numbers.

## LEVEL - II

1. If  $x = 2$  and  $x = 3$  are roots of the equation  $3x^2 - 2kx + 2m = 0$  find the value of k and m.

2. Solve the equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, x \neq 0, x \neq -1$$

3. Solve the equation  $2x^2 - 5x + 3 = 0$  by the method of completing square.

4. Using quadratic formula, solve the equation:  $p^2x^2 + (p^2 - q^2)x - q^2 = 0$ .

5. 300 apples are distributed equally among a certain number of student's .Had there been 10 more students, each would have received one apple less. Find the number of students.

6. Find the roots of Quadratic equation  $16x^2 - 24x - 1 = 0$  by using the quadratic formula.

7. Find the discriminant of the Quadratic equation  $2x^2 - 4x + 3 = 0$  and hence find the nature of its roots.

## LEVEL – III

1. In a class test, the sum of Shefali's marks in math's and English is 30. Had she got 2 marks more in math's and 3 marks less in English, the product of their marks would have been 210. Find her marks in two subjects.

2. A two digit number is such that the product of its digit is 35. When 18 is added to the number, the digits interchange the places. Find the number.

3. Solve  $3x^2 - 23x - 110 = 0$

4. Solve the following equation for 'x',  $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

5. If the roots of the equation  $(a-b)x^2 + (b-c)x + (c-a) = 0$  are equal, prove that  $2a = b+c$ .

## Self-Evaluation

1. Find the value of p so that the equation  $3x^2 - 5x + 2p = 0$  has equal roots. Also find the roots.

2. The sum of two numbers is 15. If the sum of their reciprocals is  $\frac{3}{10}$ , find the two numbers.

3. Find the quadratic equation whose roots are  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$ .

4. A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour Programme by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour Programme.
5. Divide 29 into two parts so that the sum of squares of the parts is 425.
6. Solve for x:  $9x^2 - 6ax + (a^2 - b^2) = 0$
7. If the equation  $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$  has equal roots, show that  $c^2 = a^2(1 + m^2)$

### **VALUE Based Questions**

Q1. If the price of petrol is increased by Rs. 2 per liter, a person had to buy 1 liter less petrol for Rs. 1740. Find the original price of the petrol at that time.

(a) Why do you think the price of petrol is increasing day by day?

(b) What should we do to save petrol?

2. Ramesh wants to design a rectangular park of perimeter 80 m and area  $400 \text{ m}^2$  for jogging and morning walk for the people of his colony. Is it possible to design the park? If so find the length and breadth of the park. Which value of Ramesh is depicted here?

## Answer

### LEVEL-I

1. 2
2.  $\frac{-b \pm \sqrt{D}}{2a}$
3. 5
4. -4
5. -2
6.  $k \geq 3$  or  $k \leq -3$
7.  $-1/2$
8.  $K=ab$
9.  $x^2+4x-60=0$
10. -2, 5
11. 21, 23

### LEVEL-II

1.  $K = \frac{15}{2}, m = 9$
2.  $x = \frac{3}{2}$  or  $x = \frac{-5}{2}$
3.  $x = \frac{3}{2}$  or  $x = 1$
4.  $x = -1$ , or  $x = \frac{q^2}{p^2}$
5. 50
6.  $\frac{3+\sqrt{10}}{4}, \frac{3-\sqrt{10}}{4}$
7.  $D = -8 < 0$  it has no real roots.

### LEVEL- III

1. ( Marks in maths = 12 , marks in English =18) or (marks in maths = 13 , marks in English = 17 )
2. 57
3.  $-10/3, 11$
4.  $\frac{2a+b}{3}, \frac{a+2b}{3}$

### SELF EVALUATION

1. 25/24
2. (10, 5) or (5, 10)
3.  $x^2 - 4x + 1 = 0$
4. 20 days.
5. (16, 13) or (13, 16)
6.  $(a+b)/3, (a-b)/3$

### VALUE BASED QUESTIONS

1. Rs 58 per liter
2. Yes,  $l=20m$  and  $b=20m$ .

# ARITHMETIC PROGRESSION

## KEY CONCEPT

- AN AP is a list of number in which difference of a term and the preceding term is always constant. The constant is called common difference (d) of AP.  $d = a_{n+1} - a_n$
- If a is the first term and 'd' is the common difference of an AP, then the AP is a, a+d, a+2d, a+3d.....
- The nth term of an AP is denoted by  $a_n$   
 $a_n = a + (n-1)d$  where a=first term and d= common difference  
n= number of term
- nth term from the end =  $l - (n-1)d$   
Where l=last term
- Various terms in an AP can be chosen in following manner.

No. of terms	terms	common difference
3	a-d, a, a+d	d
4	a-3d, a-d, a+d, a+3d	2d
5	a-2d, a-d, a, a+d, a+2d	d

- sum of first n natural number is  $n(n+1)/2$
- the sum of n terms of an AP with first term a and common difference d is denoted by  
 $S_n = n/2 \{2a + (n-1)d\}$   
 $s_n = n/2 (a+l)$   
 $a_n = S_n - S_{n-1}$

## LEVEL-I

1. Write fourth term of an AP if its nth term is  $3n+2$ .
2. Find A.P Which fifth term is 5 and common difference is - 3.
3. Determine the 10<sup>th</sup> term from the end of the A.P 4,9,14.....254
4. Find whether 0 is a term of the A.P 40, 37, 34, 31 .....
5. Write the value of x for which  $x+2$  ,  $2x$  ,  $2x+3$  are three consecutive terms of an A.P
6. Find the sum of first 24 term of AP 5,8,11,14.....
7. Which term of the A.P 12,7,2-3..... is -98
8. The nth term of an A.P is  $3n+5$  find its common difference.
9. Write the next term of A.P  $\sqrt{2}, \sqrt{18}$ .
10. If  $4/5$  , a, 2 three consecutive term of an A.P then find A

## LEVEL- II

11. Find the middle term of A.P 6,13,20,.....216
12. The 6<sup>th</sup> term of an A.P is -10 and its 10<sup>th</sup> term is -26. Determine the 15<sup>th</sup> term of an A.P
13. The 8th term of an A.P is 0 prove that its 38<sup>th</sup> term is triple its 18<sup>th</sup> term.
14. The sum of three numbers in A.P is 21 and their product is 231 find the numbers.
15. Find the sum of 25<sup>th</sup> term of an AP which nth term is given by  $t_n = (7-3n)$
16. Find the sum of all two digit odd positive numbers
17. Find the sum of three digits numbers which are divisible by 11
18. The sum of first 6 term of A.P is 42. The ratio its 10<sup>th</sup> term to 38<sup>th</sup> term is 1:3. Calculate the first and 13<sup>th</sup> term of the A.P
19. How many term of the A.P 17, 15, 13, 11..... must be added to get the sum 72? Explain the double answer.
20. The sum of n, 2n, 3n term of an A.P are S1, S2, and S3 respectively.



Prove that  $S_3 = 3(S_2 - S_1)$

### **LEVEL - III**

21. If in an A.P the sum of first  $m$  term =  $n$  and the sum of  $1^{st}$   $n$  term =  $m$ , then Prove that sum of  $(m+n)$  term is –  $(m+n)$
22. If  $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$  is the A.M between  $a$  and  $b$  find the value of  $n$ .
23. If the  $p^{th}$ ,  $q^{th}$ ,  $r^{th}$  term of an A.P be  $a, b, c$  respectively then show that  $a(q-r)+b(r-p)+c(p-q) = 0$
24. A man saved Rs 32 during first year Rs. 36 in second year and in this way he increases his saving by Rs. 4 every year find in what time his saving will be Rs 200
25. Find the sum of the following.  
 $(1 - \frac{1}{n}) + (1 - \frac{2}{n}) + (1 - \frac{3}{n}) + \dots$  upto  $n$  terms

### **SELF EVALUATION**

26. Find the value of  $x$  for A.P,  $1+6+11+16+\dots+x=148$
27. A man repays a loan for Rs 3250 by paying Rs 20 in the first month and then increases the payments Rs15 every month. How long will it take him to clear the loan?
28. If the sum of  $m$  terms of an A.P is the same as the sum of its  $n$  terms. Show that the sum of its  $(m+n)$  term is zero.
29. Is 51 a term of the A.P,  $5, 8, 11, 14, \dots$
30. If the  $m^{th}$  term of an A.P is  $1/n$  and  $n^{th}$  term is  $1/m$  then show that sum of  $mn$  term is  $1/2(mn+1)$ .
31. If  $2x, x+10, 3x+2$  are in A.P find the value of  $x$ .
32. Find the sum of all 3-digits numbers which are multiple of 7.
33. In an A.P the sum of first  $n$  terms is  $(3n^2/2 + 5n/2)$ . Find its  $25^{th}$  term.
34. The first term of an A.P is  $-7$  and common difference is  $5$ . Find its  $18^{th}$  term and the general term.
35. Determine the  $10^{th}$  term from the end of the A.P.  $4, 9, 14, \dots, 254$ .

### **VALUE BASED QUESTIONS**

36. A sum of Rs 700 is to be used to give 7 cash prizes to the students of a school for their overall academic performance, punctuality, regularity, cleanliness, confidence and creativity and discipline. If each prize is Rs20 less than its preceding prize. Find the value of each of the prizes.  
I) which value according to you should be awarded with maximum amount. Justify your answer.

# CO-ORDINATE GEOMETRY

## BASIC CONCEPTS

### 1. Distance Formula:-

The distance between two points A(x<sub>1</sub>,y<sub>1</sub>) and B (x<sub>2</sub>,y<sub>2</sub>) is given by the formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

COROLLARY:-The distance of the point P(x,y) from the origin O(0,0) is given by

$$OP = \sqrt{(X-0)^2 + (Y-0)^2} \quad \text{ie } OP = \sqrt{X^2 + Y^2}$$

### 2. Section Formula :-

The co-ordinates of the point P(x, y) which divides the line segment joining A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>,y<sub>2</sub>) internally in the ratio m:n are given by .

$$x = \frac{mx_2 + nx_1}{m+n} \quad y = \frac{my_2 + ny_1}{m+n}$$

### 3. Mid point Formula:-

If R is the mid-point, then m<sub>1</sub>=m<sub>2</sub> and the coordinates of R are

$$R \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

### 4. Co-ordinates of the centroid of triangle:-

The co-ordinates of the centroid of a triangle whose vertices are P(x<sub>1</sub>,y<sub>1</sub>), Q(x<sub>2</sub>,y<sub>2</sub>) and R(x<sub>3</sub>,y<sub>3</sub>) are

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

### 5. Area of a Triangle:-

The area of the triangle formed by the points P(x<sub>1</sub>,y<sub>1</sub>), Q(x<sub>2</sub>,y<sub>2</sub>) and R(x<sub>3</sub>,y<sub>3</sub>) is the numerical value of the expression.

$$\text{ar}(\Delta PQR) = \frac{1}{2} \left[ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right]$$

## **LEVEL-I**

1. Find the distance between the points P (7, 5) and Q (2, 5).
2. If  $P(\frac{a}{3}, 4)$  is the midpoint of the line segment joining the points Q(-6, 5) and R (-2,3), then find the value of a.
3. A line intersects y –axis and x-axis at the points P and Q respectively. If (2,-5) is the mid point of PQ, then find the coordinates of P and Q respectively.
4. If the distance between the points (4, p) & (1, 0) is 5, then find the value of p.
5. If the point A (1, 2), B (0, 0) and C (a,b) are collinear, then find their relation between a and b.
6. Find the rational number which the y-axis divides the segment joining (-3, 6) and (12,-3).
7. Find the coordinates of a point A, where AB is diameter of a circle whose Centre is (2,-3) and B is (1,4).
8. Find the centroid of triangle whose vertices are (3, -7), (-8,6) and (5, 10).

## **LEVEL-II**

1. If A (-2, 4), B (0,0), C(4,2) are the vertices of a  $\Delta ABC$ , then find the length of median through the vertex A.
2. Find the value of x for which the distance between the points P (4,-5) and Q(12, x) is 10 units.
3. If the points A(4, 3) and B(x, 5) are on the circle with Centre O (2, 3) then find the value of x.
4. What is the distance between the point A (c, 0) and B(0,-c)?
5. For what value of p, are the points (-3, 9), (2,p) and (4,-5) collinear?
6. Show that the points (3,2), (0,5), (-3, 2) and (0,-1) are the vertices of a square.
7. Point P divides the line segment joining the points A (2, 1) and B (5,-8) such that AP: AB=1:3  
If P lies on the line  $2x-y+k=0$ , then find the value of k.
8. Find the relation between x and y if the points (2,1), (x,y) and (7, 5) are collinear.

### **LEVEL-III**

1. Find the ratio in which the line  $2x+3y=10$  divides the line segment joining the points (1, 2) and (2, 3).
2. Prove that (4,-1), (6,0), (7,2) & (5,1) are the vertices of a rhombus. Is it a square?
3. Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are (0,-1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
4. Determine the ratio in which the point P (a,-2) divides the line joining of points (-4, 3) and B (2, -4). Also find the value of a.
5. If the point C (-1, 2) divides internally the line segment joining A (2, 5) and in the ratio 3:4. Find the Co-ordinates of B.
6. Show that points (1,1), (4,4), (4,8) and (1,5) are the vertices of a parallelogram.
7. Find the value of p, for which the points (-1, 3), (2, p) & (5,-1) are collinear.
8. If the points (-1, 3), (1,-1) and (5, 1) are the vertices of a triangle. Find the length of the median through the first vertex.

### **SELF EVALUATION**

1. Find the Centre of a circle passing through the points (6,-6), (3, 7) and (3, 3).
2. If the distance between the points (3, 0) and (0, y) is 5 units and y is positive, what is the value of y?
3. If the points (x, y), (-5,-2) and (3,-5) are collinear, then prove that  $3x+8y+31=0$ .
4. Find the ratio in which the Y-axis divides the line segment joining the points (5, -6) and (-1,-4). Also find the coordinates of the point of division.
5. By distance formula, show that the points (1,-1), (5, 2) and (9, 5) are collinear.
6. Show that the three points (a, a), (-a, -a) &  $(-a\sqrt{3}, a\sqrt{3})$  are the vertices of an equilateral triangle.

### **Board Questions**

- Q: 1)** Find the value of k, if the point P (2, 4) is equidistant from the points (5, k) and (k, 7).  
(CBSE: 2012)
- Q: 2)** If the point A(0,2) is equidistant from the points B(3,p) and C(p,5), find p. Also find the length of AB.  
(CBSE: 2014)
- Q: 3)** Find the ratio in which the point P(x, 2) divides the line-segments joining the points A (12, 5) and B (4,-3). Also, find the value of x.  
(CBSE: 2014)

**Q:4)** If the points A (-2, 1), B (a, b) and C (4, -1) are collinear and  $a-b=1$ . Find the value of a and b.

(CBSE: 2014)

**Q: 5)** In what ratio does the point (-4, 6) divide the line segment joining the points A (-6, 10) & B (3, -8)

(CBSE: 2012)

## **ASKED QUESTIONS**

**Q.1.** Mr. Gopal aged 70 lives in his house at (4, 5). He goes to shop which is located at (5, 2) and then to a park located at (3, 6). Find the distance travelled by Mr. Gopal.

In what way will you take your grandfather to the park? What are the values you exhibit when you accompany your grandfather?

Ans= values

Care for the aged. Time management, Responsibility

**Q.2.** The coordinates of houses of Sonu and Monu are (7, 3) and (4, 3) respectively. Coordinate of their school is (2, 2). If both leave their houses at the same time in the morning and also reach school in the same time.

(i) Then who travel faster, and

(ii) Which value is depicted in the question?

Ans. (i) Sonu

(ii) Punctuality

## **ANSWER KEY**

### **LEVEL-I**

1. 5
2. -12
3. (0,-10) and (4,0)
4.  $\pm 4$
5.  $2a=b$
6.  $\frac{1}{4}$
7. (3,-10)
8. (0,3)

### **LEVEL-II**

1. 5 units
2. 1, -11
3. 2
4.  $\sqrt{2c}$
5. -1
6. Proof
7.  $K=-8$
8.  $4x - 5y - 3=0$

### **LEVEL-III**

1. 2:3
2. Proof
3. 1:4
4.  $a=2/7$
5. B(-5,-2)
6. Proof
7.  $p=1$
8. 5

### **SELF EVALUATION**

1. (24,5)
2. 4
3. Proof
4. 5:1, (0,-13/3)
5. Proof
6. Proof

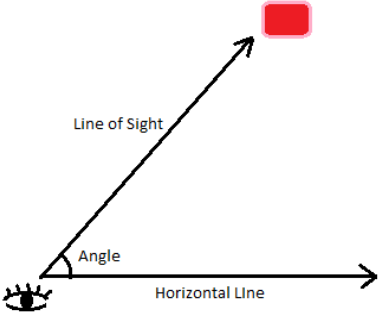
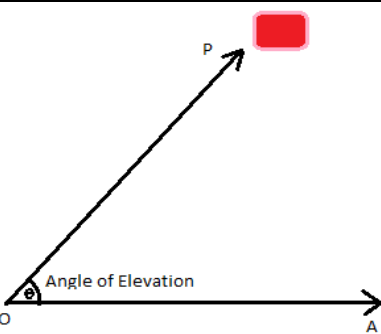
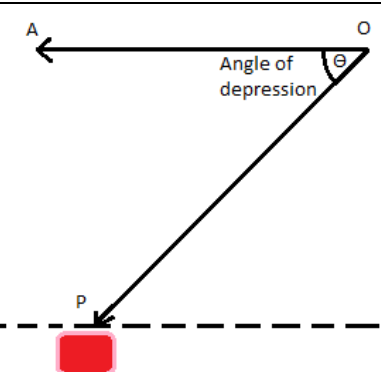
### **BOARD QUESTIONS**

1.  $K=3$
2.  $P=1, AB=\sqrt{10}$
3. 3:5,  $x=9$
4.  $a=1, b=0$
5.  $2/7$

## SOME APPLICATIONS OF TRIGONOMETRY

### HEIGHT AND DISTANCES

#### KEY POINTS

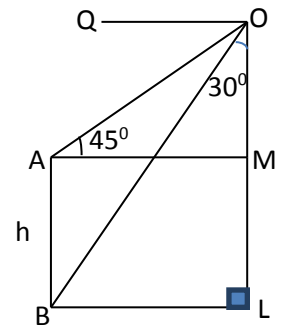
<p><b>Line of sight</b> Line segment joining the object to the eye of the observer is called the line of sight.</p>	
<p><b>Angle of elevation</b> When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.</p>	
<p><b>Angle of depression</b> When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.</p>	



## LEVEL- I

1. A pole 6cm high casts a shadow  $2\sqrt{3}$ m long on the ground, then find the sun's elevation?
2. If length of the shadow and height of a tower are in the ratio 1:1. Then find the angle of elevation.
3. An observer 1.5m tall is 20.5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.
4. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle  $60^\circ$  with the wall, find the height of the wall
5. In a rectangle ABCD,  $AB = 20\text{cm}$   $\angle BAC = 60^\circ$  then find the length of the side AD.
6. Find the angle of elevation of the sun's altitude when the height of the shadow of a vertical pole is equal to its height:
7. From a point 20m away from the foot of a tower, the angle of elevation of top of the tower is  $30^\circ$ , find the height of the tower.
8. In the adjacent figure, what are the angles of elevation and depression of the top and bottom of a pole from the top of a tower h m high:

Ans  $45^\circ, 60^\circ$



## LEVEL -II

9. The length of the shadow of a pillar is  $\sqrt{3}$  times its height. Find the angle of elevation of the source of light.
10. A vertical pole 10m long casts a shadow  $10\sqrt{3}$ m long. At the same time tower casts a shadow 90m long. Determine the height of the tower.
11. A ladder 50m long just reaches the top of a vertical wall. If the ladder makes an angle of  $60^\circ$  with the wall, find the height of the wall.
12. Two poles of height 6m and 11m stands vertically on the ground. If the distance between their feet is 12m. Find the distance between their tops.
13. The shadow of tower, when the angle of elevation of the sun is  $45^\circ$  is found to be 10m longer than when it is  $60^\circ$ . Find the height of the tower.

### **LEVEL –III**

14. The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are  $30^\circ$  and  $60^\circ$  respectively. Find the height of the tower.
15. From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are  $30^\circ$  and  $60^\circ$  respectively. Find the height of the opposite house and width of the street.
16. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be  $30^\circ$  and  $45^\circ$ . Find the height of the hill.
17. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road the angles of elevation of the top of the poles are  $60^\circ$  and  $30^\circ$ . Find the heights of pole and the distance of the point from the poles.
18. The angle of elevation of a jet fighter from a point A on the ground is  $60^\circ$ . After a flight of 15 seconds, the angle of elevation changes to  $30^\circ$ . If the jet is flying at a speed of 720km/ hr, find the constant height at which the jet is flying.
19. A window in a building is at a height of 10m above the ground. The angle of depression of a point P on the ground from the window is  $30^\circ$ . The angle of elevation of the top of the building from the point P is  $60^\circ$ . Find the height of the building.
20. A boy, whose eye level is 1.3m from the ground, spots a balloon moving with the wind in a horizontal line at same height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is  $60^\circ$ . After 2 seconds, the angle of elevation reduces to  $30^\circ$  if the speed of the wind at that moment is  $29\sqrt{3}$  m/s, then find the height of the balloon from the ground.
21. A man on the deck on a ship 14m above water level observes that the angle of elevation of the top of a cliff is  $60^\circ$  and the angle of depression of the base of the cliff is  $30^\circ$ . Calculate the distance of the cliff from the ship and the height of the cliff.
22. A tower is 50m high. It's shadow is x m shorter when the sun's altitude is  $45^\circ$  than when it is  $30^\circ$ . Find x correct to the nearest 10.

## SELF EVALUATION/HOTS

23. An airplane when flying at a height of 3125m from the ground passes vertically below another Plane at an instant when the angle of elevation of the two planes from the same point on the ground are  $30^\circ$  and  $60^\circ$  respectively. Find the distance between the two planes at that instant.
24. From the top of a building 60m high, the angels of depression of the top and bottom of a vertical lamp post are observed to be  $30^\circ$  and  $60^\circ$  respectively. Find [i] horizontal distance between the building and the lamp post [ii] height of the lamp post.
25. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h m. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are  $\alpha$  and  $\beta$ , respectively. Prove that the height of the tower is  $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$
26. The angle of elevation of a cloud from a point 60m above a lake is  $30^\circ$  and the angle of depression of the reflection of the cloud in the lake is  $60^\circ$ . Find the height of the cloud from the surface of the lake.
27. A round balloon of radius r subtends on angle  $\alpha$  at the eye of the observer whose angle of elevation of centre is  $\beta$ . Prove that the height of the Centre of the balloon is  $(r \sin \beta \cdot \operatorname{Cosec} \alpha/2)$
28. . A person standing on the bank of a river observes that the angle of elevation of top of building of an organization working for conservation of wild life. Standing on the opposite bank is  $60^\circ$ . When he moves 40m away from the bank, he finds the angle of elevation to be  $30^\circ$ . Find the height of the building and width of the river.
- (a) Why do we need to conserve the wild life?
- (b) Suggest some steps that can be taken to conserve wild life.

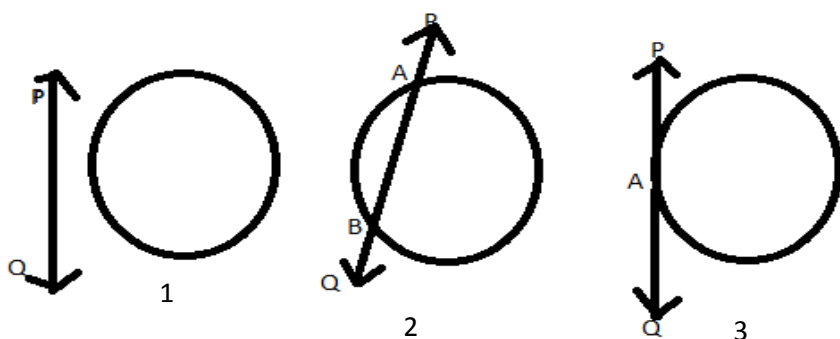
## Activities

- To make mathematical instrument eliminator ( or Sextant) for measuring the angle of elevation and depression of an object
- To Calculate the height of an object making use of Clinometer ( or Sextant)

# CIRCLES

## Key Points

1. **Circle:** A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).
2. **Secant & Tangent to a Circle:** In fig. 1 the line PQ and the circle have no common point. Line PQ is called non-intersecting. In fig. 2 line PQ a secant to a circle. In fig. 3, there is only 1 point A, which is common to the line PQ and the circle. The line is called a tangent to the circle.



## 3. Tangent to a Circle :

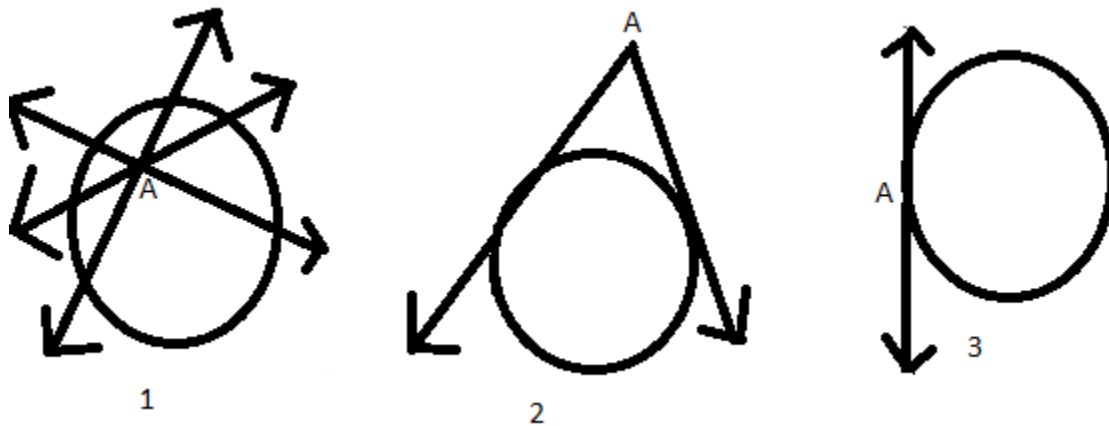
It is a line that intersects the circle at only one point. There is only one tangent at a point of the circle. The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

## 4. Theorems :

1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. The length of tangents drawn from an external point to a circle are equal.

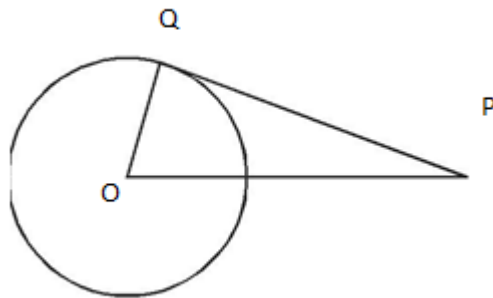
### 5. Number of tangents from a point on a circle-

- (i) There is no tangent to a circle passing through a point lying inside the circle.
- (ii) There is one and only one tangent to a circle passing through a point lying on the circle.
- (iii) There are exactly two tangents to a circle through a point lying outside the circle.

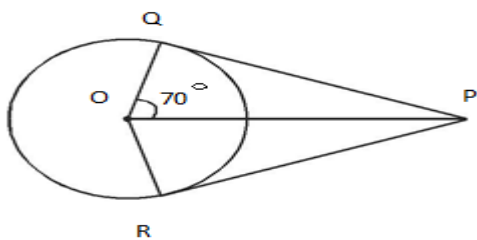


### LEVEL I

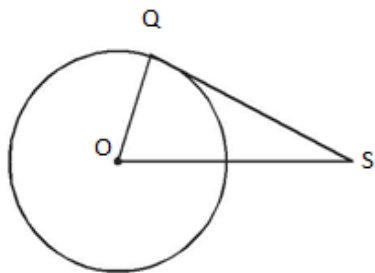
1. In the given fig. O is the centre of the circle and PQ is tangent then  $\angle POQ + \angle QPO$  is equal to



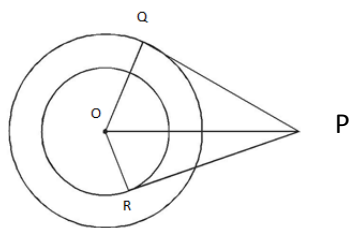
2. If PQ is a tangent to a circle of radius 5cm and  $PQ = 12$  cm, Q is point of contact, then OP is
3. In the given fig. PQ and PR are tangents to the circle,  $\angle QOP = 70^\circ$ , then  $\angle QPR$  is equal to



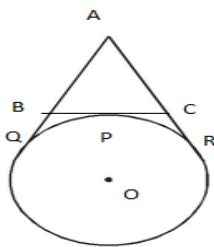
4. In the given fig.  $QS$  is a tangent to the circle,  $OS = 8$  cm,  $OQ = 6$  cm then the length of  $QS$  is



5. In the given fig  $PQ$  is tangent to outer circle and  $PR$  is tangent to inner circle. If  $PQ = 4$  cm,  $OQ = 3$  cm and  $OR = 2$  cm then the length of  $PR$  is



6. In the given fig.  $P$ ,  $Q$  and  $R$  are the points of contact. If  $AB = 4$  cm,  $BP = 2$  cm then the perimeter of  $\triangle ABC$  is

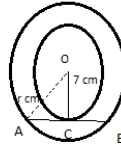
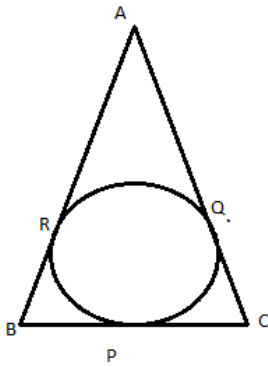


7. The distance between two tangent parallel to each other to a circle is 12 cm. The radius of circle is
8. The chord of a circle of radius 10cm subtends a right angle at its centre. Find the length of the chord.
9. How many tangents can a circle have?

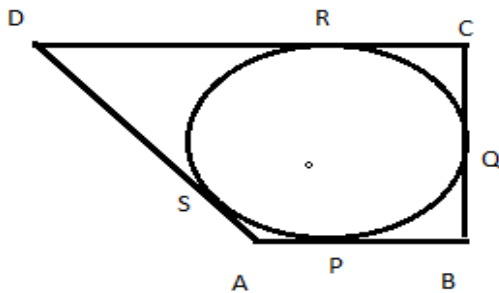
10. How many tangents can be drawn from a given point to a circle?

## LEVEL - II

11. Two concentric circles of radii  $a$  &  $b$  ( $a > b$ ) are given. Find the length of the chord of the larger circle which touches the smaller circle
12. From a point  $P$  outside the circle with centre  $O$ , tangents  $PA$  and  $PB$  are drawn to the circle. Prove that  $OP$  is the right bisector of the line segment  $AB$ .
13. A circle is inscribed in a triangle  $ABC$ , touching  $BC$ ,  $CA$  and  $AB$  at  $P$ ,  $Q$  and  $R$  respectively if  $AB = 10$  cm  $AQ = 7$  cm  $CQ = 5$  cm. Find  $BC$



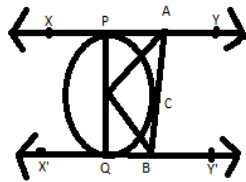
14. A quadrilateral  $ABCD$  is drawn to circumscribe a circle, as shown in the figure. Prove that  $AB + CD = AD + BC$



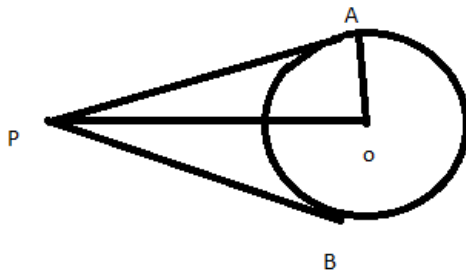
15. Two concentric circles are of radii  $7$  cm and  $r$  cm respectively, where  $r > 7$ . A chord of the larger circle of length  $46$  cm, touches the smaller circle. Find the value of  $r$ .
16. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

## LEVEL - III

17. Prove that the length of tangents drawn from an external point to a circle are equal.
18. Prove that the tangents at the extremities of any chord of a circle, make equal angle with the chord.
19. PA and PB are tangents to the circle with the centre O from an external point P, touching the circle at A and B respectively. Show that the quadrilateral AOBP is cyclic.
20. Prove that the parallelogram circumscribing a circle is a rhombus.
21. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that  $\angle AOB = 90^\circ$ .



- Q.22 Two roads starting from P are touching a circular path at A and B. Sarita runs from P to A, 20km and A to O, 15km and Reeta runs from P to O directly. (Value based question)
- Find the distance covered by Reeta.
  - Who will win the race?
  - What value is depicted by Reeta?



### SELF EVALUATION

- Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.
- Prove that perpendicular at the point of contact to the tangent to a circle passes through the centre.



3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
4. The length of a tangent from a point A at a distance 5cm from the centre of the circle is 4cm. Find the radius of the circle.                      Ans 12cm
5. Two concentric circles are of radii 6.5cm and 2.5cm. Find the length of the chord of larger circle which touches the smaller circle. Ans 3cm
6. From a point P, 10cm away from the centre of the circle, a tangent PT of length 8cm is drawn. Find the radius of the circle.                      Ans 6cm

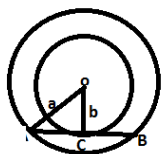
## MARKING SCHEME

### LEVEL-I

- |                   |                    |
|-------------------|--------------------|
| 1. $90^\circ$     | 2. $\sqrt{119}$ cm |
| 3. $40^\circ$     | 4. $\sqrt{28}$ cm  |
| 5. $\sqrt{21}$ cm | 6. 12 cm           |
| 7. 6 cm           | 8. $10\sqrt{2}$ cm |
9. Infinite  
10. Only 2 Tangents

### LEVEL-II

11. In Right  $\triangle ACO$ ,



$$OA^2 = OC^2 + AC^2$$

$$AC = \sqrt{a^2 - b^2}$$

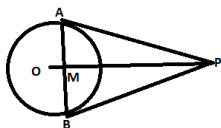
$$AB = 2AC = 2\sqrt{a^2 - b^2} \text{ [C is midpoint of AB]}$$

12. In  $\triangle MAP$  and  $\triangle MBP$ ,

$PA = PB$  [Tangents are equal]

$MP = MP$  (Common)

$\angle MPA \cong \angle MPB$  (By SAS Congruence rule)



So,  $MA = MP$  [CPCT]

And  $\angle AMP = \angle BMP$  {CP CT}

BU  $\angle AMP + \angle BMP = 180^\circ$  [Linear Pair]

$$\angle AMP = \angle BMP = 90^\circ$$

13.  $AR = AQ = 7$  cm

$$BR = (AB - AR) = (10 - 7) \text{ cm} = 3 \text{ cm}$$

$$BP = BR = 3 \text{ cm}$$

$$CP = CQ = 5 \text{ cm}$$

$$BC = BP + CP = (3 + 5) \text{ cm} = 8 \text{ cm}$$

14.  $AP = AS$ ----- (I) [Tangents from A]

$BP = BQ$ ----- (II) [Tangents from B]

$CR = CQ$ ----- (III) [Tangents from C]

$DR = DS$ ----- (IV) [Tangents from D]

$$AB + CD = (AP + BP) + (CR + DR)$$

$$= (AS + BQ) + (CQ + DS) \text{ (USING I, II, III, IV)}$$

$$= (AS + DS) + (BQ + CQ)$$

$$= AS + BC$$

Hence,  $AB + CD = AD + BC$

15.  $\triangle ACO$  we have,

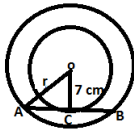
$$OA^2 = OC^2 + AC^2 \quad [\text{By Pythagoras Theorem}]$$

$$OA = \sqrt{(OC)^2 + (AC)^2}$$

$$r = \sqrt{(OC)^2 + (1/2AB)^2} \quad [C \text{ is mid-point of } AB]$$

$$r = \sqrt{7^2 + 23^2}$$

$$r = \sqrt{578}$$



$$r = 17\sqrt{2} \text{ CM}$$

### Level III

17. Correct construction

Figure

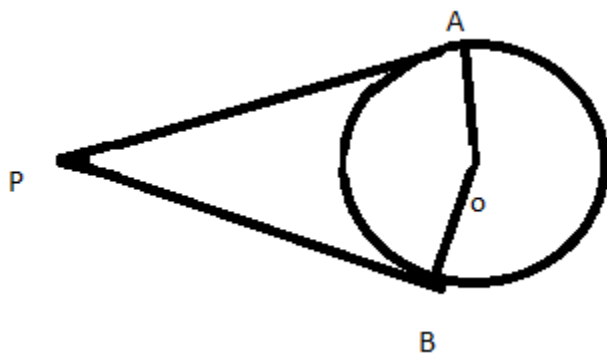
Proof

18. Correct construction

Figure

Proof

19.



Quad. OAPB,

$$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$$

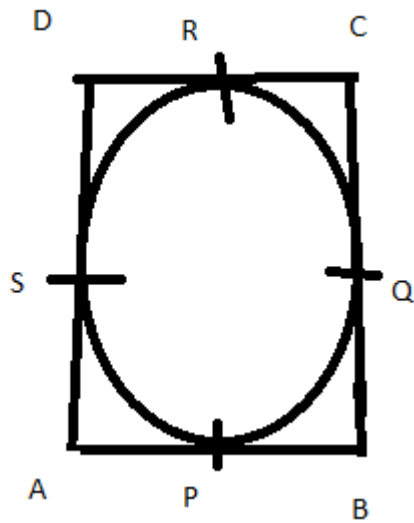
$$\text{Or, } \angle AOB + 90^\circ + \angle APB + 90^\circ = 360^\circ$$

$$\text{Or, } \angle AOB + \angle APB + 180^\circ = 360^\circ$$

$$\text{Or, } \angle AOB + \angle APB = 180^\circ$$

Hence, quad. OAPB is cyclic.

20.



$AP = AS$  .....(i) [Tangents from A]

$BP = BQ$  .....(ii) [Tangents from B]

$CR = CQ$  .....(iii) [Tangents from c]

$DR = DS$  .....(iv) [Tangents from D]

Now,  $AB + CB = AP + BP + CR + DR$

$$= AS + BQ + CQ + DS \text{ [From (i), (ii), (iii), (iv)]}$$

$$= (AS + DS) + (BQ + CQ)$$

$$= AD + BC$$

$$\text{Or, } AB + CD = AD + BC$$

$$\text{Or, } 2AB = 2AD$$

$$\text{Or, } AB = AD$$

$$\text{Hence, } AB = BC = CD = AD$$

Hence, ABCD is a rhombus.

21. In quad. APQB

$$\angle APO + \angle BQO + \angle QBC + \angle PAC = 360^\circ$$

$$\text{Or, } 90^\circ + 90^\circ + \angle QBC + \angle PAC = 360^\circ$$

$$\text{Or, } \angle QBC + \angle PAC = 180^\circ \text{ .....(i)}$$

$$\text{We have, } \angle CAO = \frac{1}{2} \angle PAC$$

$$\text{And } \angle CBO = \frac{1}{2} \angle QBC$$

$$\text{Now, } \angle CAO + \angle CBO = \frac{1}{2} (\angle PAC + \angle QBC)$$

$$= \frac{1}{2} \times 180^\circ \text{ (from eq. i)}$$

$$= 90^\circ \text{ ..... (ii)}$$

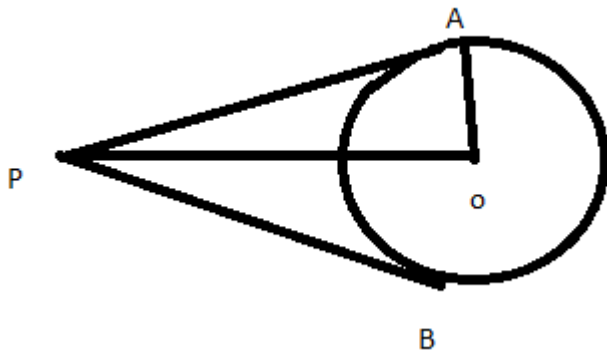
In triangle AOB,

$$\angle CAO + \angle AOB + \angle CBO = 180^\circ$$

$$\text{Or, } \angle AOB + 90^\circ = 180^\circ \text{ (from eq. ii)}$$

Or,  $\angle AOB = 90^\circ$

22.(i)



In triangle OAP,

$$OP^2 = OA^2 + AP^2 \quad (\text{By Pythagoras Theorem})$$

$$\text{Or, } OP^2 = (15)^2 + (20)^2$$

$$\text{Or, } OP^2 = 625$$

$$\text{Or, } OP = 25 \text{ km}$$

(ii) Distance covered by Rita = 25 km

$$\begin{aligned} \text{Distance covered by Sarita} &= 20 \text{ km} + 15 \text{ km} \\ &= 35 \text{ km} \end{aligned}$$

So, Rita will win the race.

(iii) Rita chooses shortest path to reach at O.

So, it shows her intelligence.

# CONSTRUCTIONS

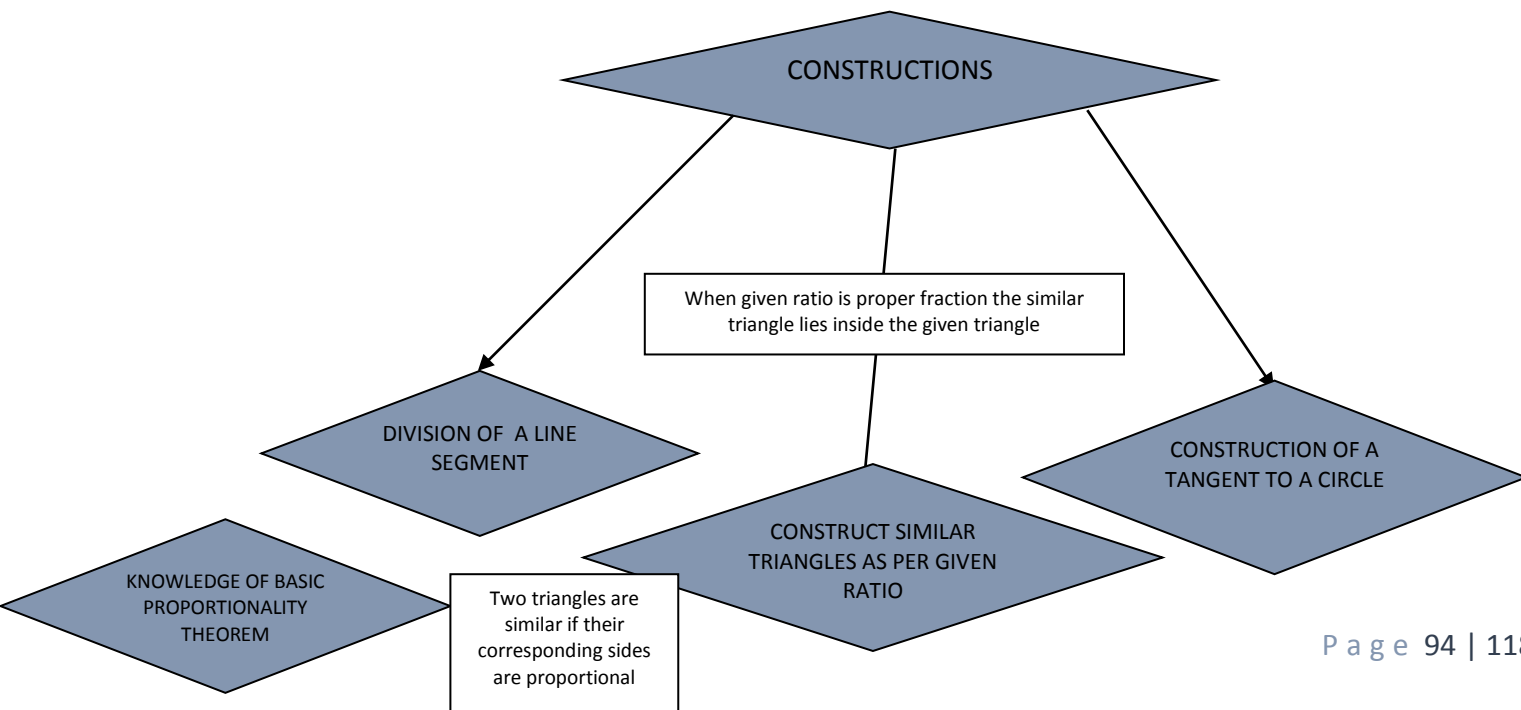
## Key Points

1. Division of a line segment in the given ratio.
2. Construction of triangles:-
  - a. When three sides are given.
  - b. When two sides and included angle given.
  - c. When two angles and one side given.
  - d. Construction of a right angled triangle.
3. Construction of triangle similar to a given triangle as per given scale factor.
4. Construction of tangents to a circle.

## EXPECTED LEARNING OUTCOMES

1. Correct use of Mathematical instruments.
2. Drawing a line segment and an angle as per the given data.
3. To divide the given line segment in the given ratio accurately.
4. Neatness and accuracy in drawing.
5. The concept of similar triangles.
6. To Construct a triangle as per the conditions given.
7. To construct similar triangle to a given triangle as per the given ratio.
8. To know that when the ratio is a proper fraction then the similar triangle lies inside the given Triangle and when improper then the similar triangle lies outside the given triangle.
9. To construct tangents to a circle from an external point given.

## CONCEPT MAP



### **LEVEL – I**

1. Draw a line segment  $AB=8\text{cm}$  and divide it in the ratio 4:3.
2. Divide a line segment of 7cm internally in the ratio 2:3.
3. Draw a circle of radius 4 cm. Take a point P on it. Draw tangent to the given circle at P.
4. Construct an isosceles triangle whose base is 7.5 cm and altitude is 4.2 cm.
5. Draw a line segment of length 9 cm. and divide it in seven equal parts.

### **LEVEL –II**

1. Construct a triangle of sides 4cm, 5cm and 6cm and then a triangle similar to it whose sides are  $\frac{2}{3}$  of the corresponding sides of the first triangle. (CBSE 2013)
2. Construct a triangle similar to a given  $\triangle ABC$  such that each of its sides is  $\frac{2}{3}$ <sup>rd</sup> of the corresponding sides of  $\triangle ABC$ . It is given that  $AB=5\text{cm}$   $BC=6\text{cm}$  and  $AC=7\text{cm}$ . Also write the steps of construction.
3. Draw a pair of tangents to a circle of radius 4cm, which are inclined to each other at an angle of  $60^\circ$ . (CBSE 2013)
4. Draw a circle of radius 5cm. From a point 8cm away from its centre construct the pair of tangents to the circle and measure their lengths.
5. Construct a triangle PQR in which  $QR=6\text{cm}$ ,  $\angle Q=60^\circ$  and  $\angle R=45^\circ$ . Construct another triangle similar to  $\triangle PQR$  such that its sides are  $\frac{5}{6}$  of the corresponding sides of  $\triangle PQR$ .
6. Draw a line segment  $AB= 7.5\text{cm}$  and locate a point P on AB such that  $AP= \frac{3}{7} AB$ . Give justification of the construction.

### **LEVEL-III**

1. Draw a circle with centre O and radius 3.5cm. Take a horizontal diameter. Extend it to both sides to point P and Q such that  $OP=OQ=7\text{cm}$ . Draw tangents PA and QB, one above the diameter and the other below the diameter. Is  $PA \parallel BQ$ .
2. Construct a  $\Delta ABC$  in which  $AB = 6\text{ cm}$ ,  $\angle A = 30^\circ$  and  $\angle B = 60^\circ$ . Construct another  $\Delta AB'C'$  similar to  $\Delta ABC$  with base  $AB' = 8\text{ cm}$ . (CBSE 2015)
3. Draw a right triangle ABC in which  $\angle B=90^\circ$ ,  $AB=5\text{cm}$ ,  $BC=4\text{cm}$ , then construct another triangle  $A'BC'$  whose sides are  $\frac{5}{3}$  times the corresponding sides of  $\Delta ABC$ . Is the new triangle also a right triangle?
4. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.
5. Draw a line segment AB of length 7 cm. Using ruler and compasses, find a point P on AB such that  $AP/AB = 3/5$ . (CBSE 2011)
6. Construct an isosceles triangle whose base is 8 cm. and altitude 4 cm. and then construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of the isosceles triangle. (CBSE 2011)
7. ABC is a right triangle in which  $AB=5.4\text{ cm}$ ,  $BC= 7\text{ cm}$  and  $\angle B = 90^\circ$ . Draw BD perpendicular on AC and a circle through B, C, D. Construct a pair of tangents from A to this circle.
8. Construct a triangle ABC in which  $AB=5\text{cm}$ ,  $\angle B=60^\circ$  and altitude  $CD=3\text{ cm}$ . Construct a triangle PQR similar to  $\Delta ABC$  such that each side of  $\Delta PQR$  is 1.5 times that of the corresponding sides of  $\Delta ABC$ .
9. Construct a tangent to a circle of radius 3.5 from a point on the concentric circle of radius 6.5 cm and measure its length. Also, verify the measurement by actual calculation.



## **Self-Evaluation**

1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5.
2. Draw an isosceles triangle ABC in which  $AB=AC=6$  cm and  $BC=5$  cm. Construct a triangle PQR similar to  $\triangle ABC$  in which  $PQ=8$  cm. Also justify the construction.
3. Two line segments AB and AC include an angle of  $60^\circ$  where  $AB=5$  cm and  $AC=7$  cm. Locate points P and Q on AB and AC respectively such that  $AP=\frac{3}{4} AB$  and  $AQ=\frac{1}{4} AC$ . Join P and Q and measure the length PQ.
4. Draw a triangle ABC in which  $AB=4$  cm,  $BC=6$  cm and  $AC=9$  cm. Construct a triangle similar to  $\triangle ABC$  with scale factor  $\frac{3}{2}$ . Justify your construction.
5. Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of  $45^\circ$ .
6. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre another circle of radius 2.5 cm. Construct tangents to each circle from the centre of the other circle.

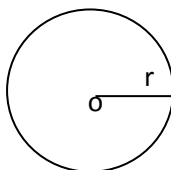
## **Value Based Question**

- (1) Two trees are to be planted at two positions A and B in the middle of a park and the third tree is to be planted at a position C in such a way that  $AC: BC= 3:4$ . How it can be done? What value is indicated from the above action?
- (2) Draw a circle of radius 5 cm. Draw tangents from the end points of its diameter. What do you observe?

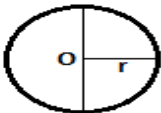
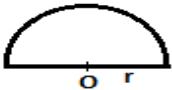
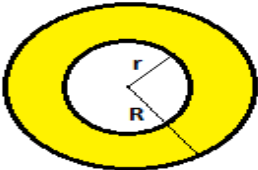


## AREAS RELATED TO CIRCLES

### KEY POINTS

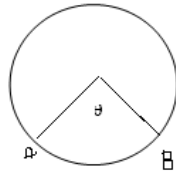
1. Circle: The set of points which are at a constant distance from a fixed point in a plane is called a circle.



2. Circumference: The perimeter of a circle is called its circumference.
3. Secant: A line which intersects a circle at two points is called secant of the circle.
4. Arc: A continuous piece of circle is called an arc of the circle.
5. Central angle: - An angle subtended by an arc at the center of a circle is called its central angle.
6. Semi-Circle: - A diameter divides a circle into two equal arcs. Each of these two arcs is called a semi-circle.
7. Segment: - A segment of a circle is the region bounded by an arc and a chord, of a circle.
8. Sector of a circle: The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.
9. Quadrant: - One fourth of a circle/ circular disc is called a quadrant. The central angle of a quadrant is  $90^\circ$ .

S.N	NAME	FIGURE	PERIMETER	AREA
1.	Circle		$2\pi r$ or $\pi d$	$\pi r^2$
2.	Semi- circle		$\pi r + 2r$	$\frac{1}{2} \pi r^2$
3.	Ring (Shaded region)		$2\pi(r + R)$	$\pi(R^2 - r^2)$
4.	Sector of a circle		$l + 2r = \frac{\pi r \theta}{180^\circ} + 2r$	$\frac{\pi r^2 \theta}{360^\circ}$ or $\frac{1}{2} l r$
5.	Area of Segment of a circle		$\frac{\pi r \theta}{180^\circ} + 2r \sin \frac{\theta}{2}$	$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta$

a. Length of an arc AB =  $\frac{\theta}{360} \times 2\pi r$



b. Area of major segment = Area of a circle – Area of minor segment

c. Distance moved by a wheel in  
1 rotation = circumference of the wheel

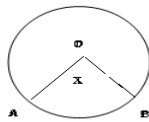
d. Number of rotation in 1 minute = Distance moved in 1 minute / circumference

### **LEVEL-I**

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is
  - i.  $\frac{22}{7}$
  - ii.  $\frac{14}{11}$
  - iii.  $\frac{7}{22}$
  - iv.  $\frac{11}{14}$
2. The area of the square that can be inscribed in a circle of 8 cm is
  - i.  $256 \text{ cm}^2$
  - ii.  $128 \text{ cm}^2$
  - iii.  $64\sqrt{2} \text{ cm}^2$
  - iv.  $64 \text{ cm}^2$
3. Area of a sector to circle of radius 36 cm is  $54\pi \text{ cm}^2$ . Find the length arc of the corresponding arc of the circle is
  - i.  $6\pi \text{ cm}$
  - ii.  $3\pi \text{ cm}$
  - iii.  $5\pi \text{ cm}$
  - iv.  $8\pi \text{ cm}$

[Ans –ii]

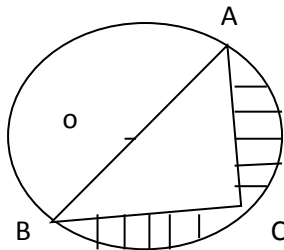
4. A wheel has diameter 84 cm. The number of complete revolution it will take to cover 792 m is.
- 100
  - 150
  - 200
  - 300
5. The length of an arc of a circle with radius 12cm is  $10\pi$  cm. The central angle of this arc is.
- $120^\circ$
  - $6^\circ$
  - $75^\circ$
  - $150^\circ$
6. The area of a circle whose circumference  $\pi$  cm is
- $11/2 \text{ cm}^2$
  - $\pi/4 \text{ cm}^2$
  - $\pi/2 \text{ cm}^2$
  - None of these
7. In figure 'o' is the centre of a circle. The area of sector OAPB is  $5/18$  of the area of the circle find x.



8. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.
9. The diameter of a cycle wheel is 21cm. How many revolutions will it make to travel 1.98km?
10. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

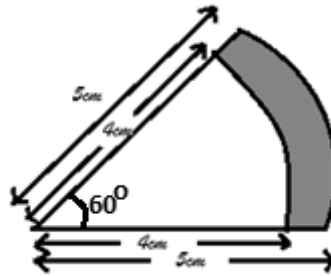
## LEVEL – II

1. Find the area of the shaded region in the figure if  $AC=24$  cm , $BC=10$  cm and o is the center of the circle (use  $\pi = 3.14$ )



2. The inner circumference of a circular track is 440m. The track is 14m wide. Find the diameter of the outer circle of the track. [Take  $\pi = 22/7$ ]

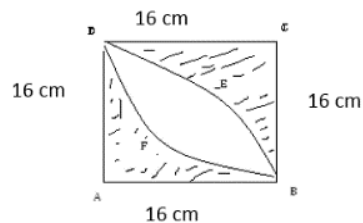
3. Find the area of the shaded region.



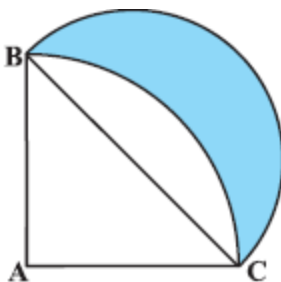
4. A copper wire when bent in the form of a square encloses an area of  $121 \text{ cm}^2$ . If the same wire is bent into the form of a circle, find the area of the circle (Use  $\pi = 22/7$ )
5. A wire is looped in the form of a circle of radius 28 cm. It is rebent into a square form. Determine the side of the square (use  $\pi = 22/7$ )

### LEVEL-III

1. Three horses are tethered with 7 m long ropes at the three corners of a triangular field having sides 20 m, 34 m 42 m. Find the area of the plot.
- Grazed by horses
  - Remains ungrazed by horses
2. Calculate the area of shaded region in given figure where ABCD is square of side 16 cm.



3. ABC is a quadrant of circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of Shaded region.



4. The length of a minor arc is  $\frac{2}{9}$  of the circumference of the circle. Write the measure of the angle subtended by the arc at the centre of the circle.
5. The area of an equilateral triangle is  $49\sqrt{3} \text{ cm}^2$ . Taking each angular point as centre, circle is drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles.  
[Take  $\sqrt{3}=1.73$ ]

### **SELF EVALUATION**

1. Two circles touch externally the sum of the areas is  $130\pi \text{ cm}^2$  and distance between their centre is 14 cm. Find the radius of circle.
2. Two circles touch internally. The sum of their areas is  $116\pi \text{ cm}^2$  and the distance between their centres is 6 cm. Find the radii of circles.
3. A pendulum swings through an angle of  $30^\circ$  and describes an arc 8.8 cm in length. Find length of pendulum.
4. The side of a square is 10 cm find the area of circumscribed and inscribed the circle.
5. An Umbrella has 8 ribs which are equally spaced. Assume Umbrella to be flat circle of radius 45 cm find the area between two consecutive ribs of umbrella.
6. Find the area of the segment AYB shown in given Fig. , If radius of the circle is 21 cm and angle  $\text{AOB} = 120^\circ$ .  
[use  $\pi = \frac{22}{7}$ ]

### **Value Based Question**

Q1. A child prepare a poster on “ save energy” on a square sheet whose each side measure 60 cm. at each corner of the sheet, she draw a quadrant of radius 17.5 cm in which she shows the ways to save energy at the centre. She draws a circle of diameter 21 cm and writes a slogan in it. Find the area of remaining sheet.  
(a) Write down the four ways by which the energy can be saved.  
(b) Write a slogan on save energy.

Q2. A birthday cake is circular in shape. This cake is equally divided among six friends where radius of the cake is 60 cm.  
i. Find the area of each piece of cake.  
ii. Which value is depicted by the friends?

## **ANSWER**

### **LEVEL-I**

1. (ii).  $14/11$
2. (ii).  $128 \text{ cm}^2$
3. (ii)  $3\pi \text{ cm}$
4. (iv) 300
5. (iv)  $150^\circ$
6. (ii)  $\pi/4$
7.  $100^\circ$
8. 36 cm
9. 6000
10.  $154/3 \text{ cm}^2$

### **LEVEL- II**

1.  $145.33 \text{ cm}^2$
2.  $D = 160 \text{ m}$
3.  $4.71 \text{ cm}^2$
4.  $154 \text{ cm}^2$
5. 44 cm

### **LEVEL- III**

1. (i)  $77 \text{ m}^2$   
(ii)  $59 \text{ m}^2$
2.  $109.7 \text{ cm}^2$
3.  $98 \text{ cm}^2$
4.  $80^\circ$
5.  $777 \text{ cm}^2$

## **SELF EVALUATION**

1. 11 cm and 3 cm
2. 4 cm and 10 cm
3. 16.8 cm
4.  $50\pi \text{ cm}^2$ ,  $25\pi \text{ cm}^2$
5.  $794.81 \text{ cm}^2$

## **VALUE BASED QUESTION**

1. Area of Remaining sheet =  $2292.19 \text{ cm}^2$   
A). Write four ways to save energy  
B). Write a slogan to save energy  
C). Write importance to save energy.
2. I) Area of each piece =  $1884 \text{ cm}^2$   
ii)  $1/6$

## **SURFACE AREA AND VOLUMES**

### **KEY CONCEPTS**

#### **1. CUBOID :**

- (I) TOTAL SURFACE AREA OF A CUBOID :  $2(LB + BH + HL)$
- (II) Volume of a cuboid =  $L \times B \times H$  sq units
- (III) Diagonal of a cuboid =  $\sqrt{L^2 + B^2 + H^2}$  units

#### **2. CUBE :**

- (I) Total Surface Area of a Cube =  $6a^2$  sq units
- (II) Volume of the Cube =  $a^3$  cubic units
- (III) Diagonal of a Cube =  $\sqrt{3}a$

#### **3. Right Circular Cylinder :**

- (I) Curved Surface Area =  $2\pi rh$
- (II) Total Surface Area =  $2\pi r(h + r)$
- (III) Volume =  $\pi r^2 h$

#### **4. Right Circular Hollow Cylinder :**

- (I) Area of each end =  $\pi(R^2 - r^2)$  [ R and r be the external radius and internal radius ]
- (II) Curved Surface Area of Hollow Cylinder =  $2\pi h(R + r)$
- (III) Total Surface Area =  $\pi(R + r)[2h + R - r]$
- (IV) Volume of material =  $\pi h(R^2 - r^2)$

#### **5. Sphere :**

- (I) Surface Area =  $4\pi r^2$
- (II) Volume =  $\frac{4}{3}\pi r^3$

#### **6. Hemisphere :**

- (I) Curved Surface Area =  $2\pi r^2$
- (II) Total Surface Area =  $3\pi r^2$
- (III) Volume =  $\frac{2}{3}\pi r^3$

#### **7. Right Circular Cone :**

- (I) Curved Surface Area =  $\pi rl$  [ l = Slant Height ]
- (II) Total Surface Area =  $\pi r(L + r)$  sq units
- (III) Volume =  $\frac{1}{3}\pi r^2 h$



## 8. Frustum of a Cone :

- (I) Volume of a Frustum of a Cone =  $\frac{\pi h (R^2 + r^2 + Rr)}{3}$   
[R – radius of base, r – radius of frustum ]
- (II) Lateral Surface Area of the Frustum of a cone =  $\pi L ( R + r )$  [ where  $L^2 = h^2 + ( R - r )^2$  ]
- (III) Total Surface Area of the Frustum of the cone =  $\pi [ R^2 + r^2 + L ( R + r ) ]$  sq units

## LEVEL WISE QUESTIONS

### LEVEL-I

1. The Surface Area of a Sphere is  $616 \text{ cm}^2$ . Find its radius.
2. The slant height of the frustum of a cone is 5 cm. if the difference between the radii of its two circular ends is 4cm, write height of the frustum.
3. A cylinder and a cone area of the same base radius and of the same height. Find the ratio of the cylinder to that of the cone.
4. Two cones have their heights in the ratio 1:3 and radii 3:1. What is the ratio of their volumes?
5. The radii of two cones are in the ratio 2:1 and their volumes are equal. What is the ratio their heights?
6. The diameter of a sphere is 6 cm. it is melted and drawn into a wire of diameter 2mm. Find the length of the wire.
7. Find the curved surface area of a right circular cone of height 15cm and base diameter is 16 cm.
8. Find the maximum volume of a cone that can be out of a solid hemisphere of radius r.
9. The diameter of the ends of a frustum of a cone are 32cm and 20 cm. If its slant height is 10 cm. Find the lateral surface area.

### LEVEL-II

1. Metallic sphere of radii 6cm, 8cm and 10cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.
2. A 20m deep well with diameter 7m is dug and the earth from digging is evenly spread out to form a platform 22m by 14m. Find the height of the platform.
3. Two cubes of volume  $64 \text{ cm}^3$  are joined end to end. Find the volume of the sphere.
4. The largest sphere is curved out of a cube of a side 7cm. Find the volume of the sphere.
5. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105m and the slant height of the conical part is 53m. Find the total canvas used in making the tent.
6. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14cm and the total height of the vessel is 13 cm. Find it's capacity?
7. A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2cm and the height of the cylindrical and conical position are 12cm and 7cm respectively. Find the volume of the solid toy.

8. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of  $\pi$ .

### **Level-III**

1. A hemispherical depression is cut from one face of the cubical wooden block such that the diameter  $l$  of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.
2. A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass was 10cm, find what the apparent capacity of the glass was and what the actual capacity was.
3. The height of a cone is 30cm. A small cone is cut off at the top by a plane parallel to the base of its volume be  $\frac{1}{27}$  of the volume of the given cone, at what height above the base is the section made ?
4. An oil funnel of tin sheet consists of a cylindrical portion 10cm long attached to 4 frustum of a cone. If the total height be 22cm, diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be 18cm. Find the area of the tin required to make the funnel.
5. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2cm and the total height of the toy is 10.2cm. Find the volume of the wooden toy.

### **SELF-EVALUATION**

1. A tent is of the shape of a right circular cylinder up to a height of 3m and then becomes a right circular cone with a maximum height of 13.5m, above the ground.  
Calculate the cost of painting the inner side of the tent at the rate of Rs. 2 per sq. metre, if the radius of the edge is 14 metres.  
[ Total Area =  $1034\text{m}^2$ , Cost of painting = Rs. 2068]
2. A bucket is in the form of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28cm and 21cm respectively. Find the height of the bucket.  
[Height of the bucket = 15cm]
3. The perimeter of the ends of a frustum are 48cm and 36cm. If the height of the frustum be 11cm, find its volume.  
[1553  $\text{cm}^3$ ]
4. If the radii of the circular ends of a conical bucket which is 45cm high, are 28cm and 7cm. Find the capacity of the bucket.  
[Capacity of the bucket =  $48510\text{cm}^3$ ]
5. A pen stand made of wood is in the shape of a cuboid with four conical depression's to hold pens. The dimensions of the cuboid are 15cm by 10 cm by 3.5cm. The diameter of each of the depression is 1cm and the depth is 1.4 cm. Find the volume of the word in the entire stand.  
[ans. 523.53  $\text{cm}^3$ ]
6. Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.  
[ans. 350  $\text{cm}^2$  ]
7. The diameter of a metallic sphere is 6cm. The sphere is melted and drawn into a wire of uniform cross-section. If the length of the wire is 36m. Find its radius.

[ans 10 mm ]

8. If the diameter of cross-section of a wire is decreased by 5%. How much percent will the length be increased so that the volume remains the same?

[ans. 10.8% ]

# PROBABILITY

## KEY POINTS

1. **Probability:** - The theoretical probability of an event E, written as P (E) is defined as.  
$$P(E) = \frac{\text{Number of outcomes Favorable to E}}{\text{Number of all possible outcomes of the experiment}}$$
  
Where we assume that the outcomes of the experiment are equally likely.
2. The probability of a sure event (or certain event) is 1.
3. The probability of an impossible event is 0.
4. The probability of an Event E is number P (E) such that  $0 \leq P(E) \leq 1$ .
5. Elementary events: - An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
6. For any event E,  $P(E) + P(\bar{E}) = 1$ , where  $\bar{E}$  stands for not E, E and  $\bar{E}$  are called complementary event.
7. Performing experiments:-
  - a. Tossing a coin.
  - b. Throwing a die.
  - c. Drawing a card from deck of 52 cards.
8. **Sample space:**-The set of all possible outcomes in an experiment is called sample space.
9. An event is a subset of a sample space.
10. Equally likely events - If one event cannot be expected in preference to other event then they are said to be equally likely.

## LEVEL-I

1. The probability of getting bad egg in a lot of 400 is 0.035. Then find the no. of bad eggs in the lot.
2. Write the probability of a sure event.
3. What is the probability of an impossible event?
4. When a dice is thrown, then find the probability of getting an odd number less than 3.
5. A girl calculates that the probability of her winning the third prize in a lottery is 0.08. If 6000 tickets are sold, how many ticket has she bought.
6. What is probability that a non-leap year selected at random will contain 53 Sundays.
7. A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is  $\frac{11}{20}$  and that of blue ball is  $\frac{1}{5}$ , then what is the no. of black ball?
8. Two coins are tossed simultaneously. Find the probability of getting exactly one head.
9. A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting an ace.
10. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.

## LEVEL-II

1. Find the probability that a no. selected at random from the number 3, 4, 5, 6...25 is prime.
2. A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (a) of red colour (b) not of green colour.
3. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing  
(a) A face card (b) card which is neither a king nor a red card
4. A dice is thrown once. What is the probability of getting a number greater than 4?
5. Two dice are thrown at the same time. Find the probability that the sum of two numbers appearing on the top of the dice is more than 9.
6. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice.
7. A coin is tossed two times. Find the probability of getting almost one head.
8. Cards with numbers 2 to 101 are placed in a box. A card selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square.
9. Find the probability of getting the letter M in the word "MATHEMATICS".

### **LEVEL-III**

1. Cards bearing numbers 3, 5... 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing (a) a prime number less than 15 (b) a number divisible by 3 and 5.
2. Two dice are thrown at the same time. Find the probability of getting (a) same no. on the both side (b) different no. on both dices.
3. A child game has 8 triangles of which three are blue and rest are red and ten squares of which six are blue and rest are red. One piece is lost at random. Find the probability of that is (a) A square (b) A triangle of red colour.
4. Two dice are thrown simultaneously. What is the probability that:  
(a) 5 will not come up either of them? (b) 5 will come up on at least one? (c) 5 will come at both dice?
5. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (a) heart (b) queen (c) clubs
6. A game consist of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., 3 heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.
7. Cards bearing numbers 1, 3, 5... 37 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing

(a) A prime number less than 15

(b) a number divisible by 3 and 5.

8. A dice has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and total score is

recorded. (a) how many different scores are possible? (b) What is the probability of getting a total of seven?

### **Self-Evaluation/HOTS**

1. Two dice are thrown simultaneously. Find the probability of getting an even number as the sum.
2. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is:
  - (i) An even number
  - (ii) A number less than 14
  - (iii) A number is perfect square
  - (iv) A prime number less than 20
3. Out of the families having three children, a family is chosen random. Find the probability that the family has
  - (i) Exactly one girl
  - (ii) At least one girl
  - (iii) At most one girl

### **Value based Question**

Q1. In a survey, it was found that 40 % people use petrol, 35 % uses diesel and remaining uses CNG for their vehicles. Find the probability that a person uses CNG at random.

(a) Which fuel out of above 3 is appropriate for the welfare of the society?

Board questions of previous years

### **Level -I**

1. A die is thrown once. What is probability of getting a number greater than 4?
2. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. Find the probability of getting a black ball?
3. A die is thrown once. Find the probability of getting.
  - a) prime number
  - b) A number divisible by 2.

### **Level -II**

1. A bag contains card which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears.

- a.) A Two digit number
  - b.) A number which is perfect square.
2. Two dice are rolled once. Find the probability of getting such numbers on the two dice whose product is 12.

### **Level – III**

1. Red queens and black jacks are removed from a pack of 52 playing card. A card is drawn at random from the remaining card, after reshuffling them. find the probability that the drawn card is:
- i) King      ii) of red colour      iii) a face card      iv) queen
2. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards after reshuffling them. Find the probability that the card drawn is
- i) Of red colour      ii) a queen      iii) an ace      iv) a face card.
3. In a family of 3 children, find the probability of having a least 1 boy.
4. Three unbiased coins are thrown simultaneously. Find the probability of getting.
- i. Exactly two heads.
  - ii. At least two heads.
  - iii. At most two heads.

## **ANSWER**

### **LEVEL-I**

1. 14
2. 1
3. 0
4.  $\frac{1}{6}$
5. 480
6.  $\frac{1}{7}$
7. 10
8.  $\frac{1}{2}$
9.  $\frac{1}{13}$
10.  $\frac{2}{7}$

### **LEVEL - II**

1.  $\frac{8}{23}$
2. A.  $\frac{5}{12}$  B.  $\frac{3}{4}$
3. A.  $\frac{3}{13}$  B.  $\frac{6}{13}$
4.  $\frac{1}{3}$
5.  $\frac{1}{6}$
6.  $\frac{5}{6}$
7.  $\frac{3}{4}$
8.  $\frac{9}{100}$
9.  $\frac{2}{11}$

### **LEVEL – III**

1. A.  $\frac{5}{17}$  B.  $\frac{1}{17}$
2. A.  $\frac{1}{6}$  B.  $\frac{5}{6}$
3. A.  $\frac{5}{9}$  B.  $\frac{5}{18}$
4. A.  $\frac{25}{36}$  B.  $\frac{11}{36}$  C.  $\frac{1}{36}$
5. A.  $\frac{13}{49}$  B.  $\frac{3}{49}$ , C  $\frac{10}{49}$
6.  $\frac{3}{4}$
7. A.  $\frac{5}{19}$  B.  $\frac{1}{19}$
8. A. 6 scores B.  $\frac{1}{3}$



## **SELF EVALUATION**

1.  $\frac{1}{2}$
2. A.  $\frac{1}{2}$  B  $\frac{3}{25}$  C  $\frac{9}{100}$ , D.  $\frac{2}{25}$
3. A.  $\frac{1}{5}$ , B. (i)  $\frac{1}{4}$  (ii) 0

## **VALUE BASED QUESTION**

1. Probability = 0.25  
CNG

## **BOARD QUESTION**

### **LEVEL-I**

1.  $\frac{1}{3}$
2.  $\frac{3}{5}$
3.  $\frac{1}{2}$ ,  $\frac{1}{2}$

### **LEVEL- II**

1.  $\frac{81}{89}$ ,  $\frac{8}{89}$
2.  $\frac{1}{9}$

### **LEVEL- III**

1.  $\frac{1}{12}$ ,  $\frac{11}{48}$ ,  $\frac{1}{6}$ ,  $\frac{1}{24}$
2.  $\frac{10}{23}$ ,  $\frac{1}{23}$ ,  $\frac{2}{23}$ ,  $\frac{3}{23}$
3.  $\frac{7}{8}$
4.  $\frac{3}{8}$ ,  $\frac{1}{2}$ ,  $\frac{7}{8}$

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केन्द्रीय विद्यालय संगठन, पटना संभाग

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

संकलित परीक्षा-II

Summative Assessment-II (2016-17)

कक्षा—दशम्

विषय : गणित

Class: - X

Sub:-Mathematics

निधारित समय : 3 घंटे

अधिकतम अंक: 90

Time Allowed:-3 Hrs. Max<sup>m</sup>. Marks:-90

सामान्य निर्देश :

- 1) सभी प्रश्न अनिवार्य 7 हैं।
- 2) इस प्रश्न पत्र में 31 प्रश्न हैं, जो चार खण्डों अ, ब, स और द में विभाजित हैं।
- 3) खण्ड अ में 1—1 अंक वाले चार प्रश्न हैं। खण्ड ब में 6 प्रश्न हैं जिनमें से प्रत्येक 2 अंक का है। खण्ड स में 10 प्रश्न 3—3 अंकों के हैं। खण्ड द में 11 प्रश्न हैं जिनमें से प्रत्येक 4 अंक का है।
- 4) कैलकुलेटर का प्रयोग वर्जित है।

### General Instructions:-

(1) General Instructions:

1. All questions are compulsory.

2. The question Paper consists of 31 questions divided into 4 sections A, B, C and D. Section-A contains 4 questions of 1 mark each, Section-B contains 6 questions of 2 marks each, Section-C contains 10 questions of 3 marks each and Section-D contains 11 questions of 4 marks each.

3 .Use of Calculator is not permitted

खण्ड—अ

### Section-A

- प्रश्न-(1). अगर  $ax^2+bx+c=0$  का मूल समान है तो  $c$  का मान निकालें।  
Q.No. (1) If  $ax^2+bx+c=0$  has equal roots. Find the value of  $c$ .
- प्रश्न-(2). धरती पर एक मीनार खड़ी है। धरती के एक बिन्दु से, जो मीनार के एक बिंदु से 20 मीटर दूर है, मीनार के शिखर का उन्नयन कोण  $30^\circ$  डिग्री है तो मीनार की ऊंचाई ज्ञात करें।  
Q.No. (2) A tower stands on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$ . Find the height of the tower.
- प्रश्न-(3) अच्छी प्रकार से मिलायी गयी 52 तास के पत्ते की एक गद्दी में से एक पत्ता निकला जाता है / ईट की बेगम आने की प्रायिकता क्या होगी।  
Q.No. (3) one card is drawn from a well shuffled deck of 52 cards. Find the probability of getting the queen of diamond.
- प्रश्न-(4) यदि वृत्त की दो त्रिज्याओं के बीच का कोण  $130^\circ$  डिग्री हो तो, इन त्रिज्याओं के सिरों पर खींची गयी स्पर्श रेखाओं के बीच का कोण क्या होगा।  
Q.No. (4) If angle between two radii of a circle is  $125^\circ$ . Find the angle between the tangents at the ends of the radii.

### खण्ड—ब

#### Section-B

- प्रश्न-(5) यदि त्रिज्या 3 सेंटीमीटर वाले वृत्त की दो स्पर्श रेखाएँ ऐसी खींची जाये की उनके बीच का कोण  $60^\circ$  हो तो प्रत्येक स्पर्श रेखा की लम्बाई ज्ञात करें ।
- Q.No. (5) If two tangents inclined at an angle  $60^\circ$  are drawn to a circle of radius 3cm, then find the length of each tangent
- प्रश्न-(6) दो सकेन्द्रीय वृत्तों के त्रिज्या 13 सेंटीमीटर एवं 12 सेंटीमीटर है । बड़े वृत्त के जीवा की लम्बाई ज्ञात करें जो छोटे वृत्त पर खींची गयी स्पर्श रेखा है ।
- Q.no. (6) Two concentric circles of radii 13 cm and 12 cm are given. Find the length of the chord of the larger circle which touches the smaller circle.
- प्रश्न-(7) द्वीघात समीकरण  $2x^2 - \sqrt{5}x + 1$ . के मूल की प्रकृति बतायें ।
- Q.No. (7) What is the nature of roots of the quadratic equation  $2x^2 - \sqrt{5}x + 1$ .
- प्रश्न-(8). a, 7, b, 23 एवं c सामानांतर श्रेणी में हैं । a, b, एवं c का मान ज्ञात करें ।
- Q.No.(8) Find a, b and c such that a, 7, b, 23, c are in A.P.
- प्रश्न-(9). बिंदु A(7,1) तथा B(3,5), बिंदु P(x, y) से समान दूरी पर है तो x तथा y के बीच सम्बंध निकालें.
- Q.No.(9) Find the relation between x and y such that the point P(x, y) is equidistant from the points A(7,1) and B(3,5).
- प्रश्न-(10). ज्ञात कीजिए कि बिंदु P (-4,x), बिंदु A(-5, -4) और B(-2,3) को मिलने वाले रेखाखंड को किस अनुपात में विभाजित करता है, तथा x का मान निकालें।
- Q.No.(10) Find the ratio in which the point P(-4,x) divides the line segment joining the points A(-5,-4) and B(-2,3) and also find the value of x.

### खण्ड—स

#### Section-C

- प्रश्न-(11). सामानांतर श्रेणी के वह तीन संख्या निकालें जिनका योग 15 तथा गुणनफल 80 है ।
- Q.No. (11) Find three numbers in A.P whose sum is 15 and the product is 80.
- प्रश्न-(12). दो वर्गों के क्षेत्रफल का योग 468 मी<sup>2</sup> है । यदि उनके परिमापों का अंतर 24 मी. है तो दोनों वर्गों के भुजाओं का द्विघात समीकरण बनावे ।
- Q.No. (12) Sum of the areas of two squares is 468 m<sup>2</sup>. If the difference of their Perimeter is 24, formulate the quadratic equation of the sides of the two squares.
- प्रश्न-(13). मीनार के आधार से और एक सरल रेखा में 4 मी. और 9 मी. की दूरी पर स्थित दो बिंदुओं से मीनार के शिखर के उन्नयन कोण पूरक कोण है । सिद्ध कीजिए कि मीनार की ऊँचाई 6 मी. है ।
- Q.No. (13) The angle of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.
- प्रश्न-(14). बिंदुओं A(2, -2) और B(3, 7) को जोड़ने वाले रेखाखंड को रेखा  $2x + y - 4 = 0$  जिस अनुपात में विभाजित करती है उसे ज्ञात कीजिए ।

Q.No.(14) Determine the ratio in which the line  $2x + y - 4 = 0$  divides the line segment joining the points A(2, -2) and B(3, 7).

प्रश्न-(15). एक पासा को एक बार फेंकते हैं / (i) 3 (ii) 4 (iii) 8 से बड़ी संख्या, प्राप्त होने की प्रायिकता क्या होगी।

Q.No. (15) A die is thrown once. What is the probability that it shows (i) 3 (ii) 4 (iii) number greater than 4.

प्रश्न- (16). 15 सेमी. त्रिज्या वाले एक वृत्त की कोई जीवा केंद्र पर  $60^\circ$  का कोण अंतरित करता है। संगत लघु एवम दीर्घ वृत्तखंडों का क्षेत्रफल ज्ञात कीजिए।

Q.No.(16) A chord of a circle of radius 15 cm subtends an angle of  $60^\circ$  at the Centre. Find the areas of the corresponding minor and major segments of the circle. [Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ ]

प्रश्न-17. पृष्ठीय क्षेत्रफल और आयतन का ज्ञान देने के लिए एक शिक्षक ने कक्षा में मिट्टी लेकर आए। उन्होंने मिट्टी से 6cm त्रिज्या तथा 24cm ऊँचाई का एक शंकु बनाया। फिर उन्होंने उस मिट्टी के शंकु को गोला में परिवर्तित कर दिया। उस गोले की त्रिज्या ज्ञात करें। क्या शिक्षण सामग्री से शिक्षण अधिगम प्रक्रिया को बढ़ावा मिलता है? अपने जवाब का औचित्य साबित करें।

Q.No.(17) A teacher brings clay in the classroom to give the concept of surface area and volume in mensuration. He makes a cone of radius 6cm and height 24 cm with the clay. Then he moulds that cone into sphere. Find the radius of sphere formed. Do teaching aids enhance the teaching learning process? Justify your answer.

प्रश्न-18. एक 14cm व्यास वाले पाईप के माध्यम से पानी 15 km/h की दर से एक घनाभाकार तालाब में जा रहा है, जो 50m लम्बा और 44m चौड़ा है। कितने समय बाद तालाब में पानी का स्तर 21cm ऊँचा हो जाएगा।

Q.No.(18) Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21cm?

प्रश्न-(19). 5 cm त्रिज्या तथा 10cm ऊँचाई वाला आइसक्रीम शंकु जिसमें आइसक्रीम भरी है। शंकु में भरे आइसक्रीम का आयतन निकालें जबकि शंकु का  $1/6$  भाग खाली है।

Q.No.(19) An ice-cream cone is full of ice cream having radius 5cm and height 10cm. Calculate the volume of ice-cream provided that its  $1/6$  part is left unfilled with ice-cream.

प्रश्न-20. एक 3cm, 4cm और 5cm किनारों वाले धातु के तीन ठोस घनों को पिघलाकर एक अकेला घन बनाया गया है। इस प्रकार बने घन का पृष्ठीय क्षेत्रफल ज्ञात करें।

Q.No.(20) Three metallic solid cubes whose edges are 3cm, 4cm & 5cm are melted and formed into a single cube. Find the surface area of cube so formed.

### खण्ड-द

### Section-D

प्रश्न-21. किसी प्रकृतिक संख्या में 12 जोड़ने पर वह अपने व्युत्क्रम का 160 गुणा हो जाता है। वह संख्या ज्ञात करें।

Q.No.(21) A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

प्रश्न-22. किसी सामानांतर श्रेणी के प्रथम सात पदों का योग 49 तथा 17 पदों का योग 249 है। प्रथम  $n$  पदों का योग निकालें।

Q.No.(22) If the sum of first 7 terms of AP is 49 and that of 17 terms is 249. Find the sum of first  $n$  terms.

प्रश्न-23. एक रेलगाड़ी 360 km की दूरी एक समान चाल के साथ तय करती है। यदि रेलगाड़ी यही दूरी 5 km/h अधिक चाल से तय करती तो यात्रा में 48 min कम समय लगता। रेलगाड़ी की प्रारम्भिक चाल ज्ञात कीजिए।

Q.No.(23) A train travelling at a uniform speed for 360 km, would have taken 48 min, less than to travel the same distance, if its speed was 5 km/h more. Find the original speed of the train.

प्रश्न-24. आकृति में XY तथा X'Y', O केंद्र वाले किसी वृत्त पर दो समांतर स्पर्श रेखाएँ हैं और स्पर्श बिंदु C पर स्पर्श रेखा AB, XY को A तथा X'Y' को B पर काटती है। सिद्ध करें कि  $\angle AOB = 90^\circ$

Q.No.(24) In figure X, Y and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that  $\angle AOB = 90^\circ$ .

प्रश्न-(25). सिद्ध करें - वृत्त से बिंदु पर खींची गई स्पर्श रेखाओं की लम्बाइयों बराबर होती हैं।

Q.No.(25) Prove that the lengths of two tangents drawn from an external point to a circle are equal.

प्रश्न-(26). एक त्रिभुज ABC बनाइए जिसमें  $BC=6\text{cm}$ ,  $AB=5\text{cm}$  और  $\angle ABC = 60^\circ$  हो। फिर एक त्रिभुज की रचना कीजिए, जिसकी भुजाएँ  $\Delta ABC$  की संगत भुजाओं की  $\frac{3}{4}$  गुणी हों।

Q.No.(26) Draw a  $\Delta ABC$  with sides  $BC=6\text{cm}$ ,  $AB=5\text{cm}$  and  $\angle ABC = 60^\circ$ . Then construct a triangle whose sides are  $\frac{3}{4}$  of the corresponding sides of  $\Delta ABC$ .

प्रश्न-27. समुद्र तल से 75m ऊँची लाईट हाउस के शिखर से देखने पर दो समुद्री जहाजों के अवनमन कोण  $30^\circ$  और  $45^\circ$  हैं। यदि लाईट हाउस के एक ही ओर एक जहाज दूसरे जहाज के ठीक पीछे हो तो दो जहाजों के बीच की दूरी ज्ञात करें।

Q.No.(27) As observed from the top of a 75m high light house from the sea level, the angle of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships.

प्रश्न-28. सभी काले फेस कार्ड को 52 पत्तों की एक गड्डी से हाटाकर अच्छी तरह फेंट कर एक कार्ड निकाला जाता है :- (i) फेस कार्ड (ii) लाल कार्ड (iii) काला कार्ड (iv) बादशाह, आने की प्रायिकता ज्ञात करें।

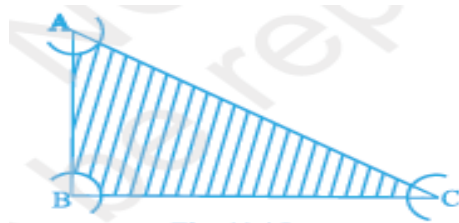
Q.No. (28) All the black face cards are removed from a pack of 52 playing cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting a –

i) face card                      ii) red card                      iii) black card                      iv) king

प्रश्न-(29). K के मान ज्ञात कीजिए, यदि बिंदु  $A(K+1, 2K)$ ,  $B(3K, 2K+3)$  और  $C(5K-1, 5K)$  संरेख हैं।

Q.No. (29) Find the values of K if the points  $A(K+1, 2K)$ ,  $B(3K, 2K+3)$  and  $C(5K-1, 5K)$  are collinear.

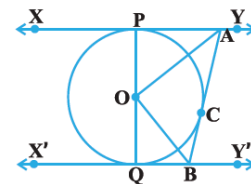
प्रश्न-30. एक त्रिभुज ABC के A, B और C शीर्षों को केंद्र मानकर तथा त्रिज्याएँ 5 cm लेकर आकृति में दिए गए चाप खींचे गए हैं। यदि  $AB=14\text{ cm}$ ,  $BC=48\text{ cm}$  और  $CA=50\text{ cm}$  है तो छायांकित क्षेत्र का क्षेत्रफल निकालें।



Q.No.(30) With the vertices A, B and C of a triangle ABC as centre, arcs are drawn with radii 5cm each as shown in fig. If  $AB = 14\text{ cm}$ ,  $BC = 48\text{ cm}$  and  $CA = 50\text{ cm}$ , then find the area of the shaded region. (Use  $\pi = 3.14$ )

प्रश्न-31. एक शंकू की ऊँचाई 30cm है। इस शंकू के आधार के समानांतर एक छोटा शंकू काटा गया जिसका आयतन दिये गए शंकू का  $\frac{1}{27}$  है; तो ज्ञात करें कि दिये गए शंकू के आधार से कितनी ऊँचाई पर नए शंकू को काटा गया।

Q.No.(31) The height of the cone is 30cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be  $\frac{1}{27}$  of the given cone, at what height above the base is the section made?



## **ACTIVITES (TERM-II)**

### **(Any Eight)**

- Activity1: To find geometrically the solution of a Quadratic Equation  $ax^2+bx+c=0$ ,  $a \neq 0$  (where  $a=1$ ) by using the method of computing the square.
- Activity2: To verify that given sequence is an A.P (Arithmetic Progression) by the paper Cutting and Paper Folding.
- Activity3: To verify that  $\sum n = \frac{n(n+1)}{2}$  by Graphical method
- Activity4: To verify experimentally that the tangent at any point to a circle is perpendicular to the Radius through that point.
- Activity5: To find the number of tangent from a point to the circle
- Activity6: To verify that lengths of tangents drawn from an external Point, to a circle are equal by using method of paper cutting, paper folding and pasting.
- Activity7: To Draw a quadrilateral similar to a given quadrilateral as per given scale factor (Less than 1)
- Activity8: (a) To make mathematical instrument clinometer (or sextant) for measuring the angle of elevation/depression of an object  
(b) To calculate the height of an object making use of clinometers (or sextant)
- Activity9: To get familiar with the idea of probability of an event through a double color card experiment.
- Activity10: To verify experimentally that the probability of getting two tails when two coins are tossed simultaneously is  $\frac{1}{4} = (0.25)$  (By eighty tosses of two coins)
- Activity11: To find the distance between two objects by physically demonstrating the position of the two objects say two Boys in a Hall, taking a set of reference axes with the corner of the hall as origin.
- Activity12: Division of line segment by taking suitable points that intersects the axes at some points and then verifying section formula.
- Activity13: To verify the formula for the area of a triangle by graphical method.
- Activity14: To obtain formula for Area of a circle experimentally.
- Activity15: To give a suggestive demonstration of the formula for the surface Area of a circus Tent.
- Activity16: To obtain the formula for the volume of Frustum of a cone.